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DEVELOPMENT OF THE MULTITURN LOOP
ANTENNA AT H. F. FOR SHIPBOARD
APPLICATIONS

P. Bohley, et al

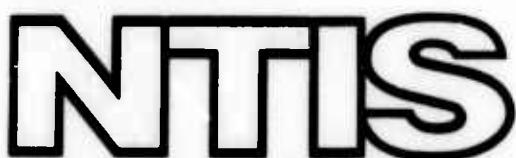
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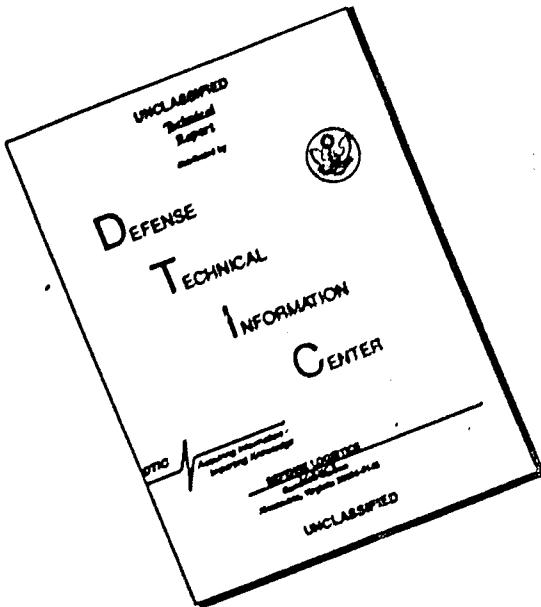
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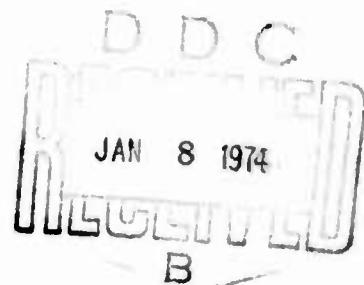
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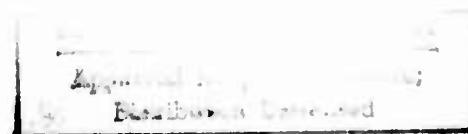
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ABSTRACT

This report describes efforts to develop the electrically small multturn loop antenna for shipboard use at H.F. A 2 foot by 2 foot by 1 foot high prototype was constructed and tested over a tuning range of from 2 to 30 MHz. Several methods for measuring antenna efficiency are described. The effect of changes of loop parameters are discussed. Midband efficiency was 30% (10 MHz) increasing above and decreasing below this frequency. Efficiency increases considerably with increasing coil diameter. Further study is recommended.

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I. INTRODUCTION

One of the most common H.F. shipboard antennas in use today is the vertical monopole, or "whip". This type of antenna has been used and studied for many years and its characteristics are well known. Its development aboard ship has in general resulted in a single, vertical 35 foot radiating element in combination with an impedance matching tuner, to correct for impedance mismatch over the entire H.F. band. This combination, particularly in view of the mechanically and electrically complicated environment usually found aboard ship, is not a totally satisfactory solution to all H.F. antenna requirements.

Recent work with loop antennas at VHF, specifically the multiturn loop antenna (MTL), has indicated the possibility of the development of the MTL as a complement to, or in some cases a replacement for, the whip/tuner combination. The most obvious advantage of the MTL over the whip/tuner combination is its small size. Efficient operation can be achieved with loops whose major dimensions are of the order of $\lambda/30$. Whips of this length usually exhibit very poor efficiency at HF due, primarily, to large currents in the tuning mechanism. With this basic advantage in mind a program was initiated to develop the MTL specifically for the shipboard environment.

Principal desirable characteristics were established as follows:

- 1) Small size/low profile
- 2) High efficiency
- 3) Tunable from 2-30 MHz
- 4) Input power to 1 kw
- 5) Simple operation.

II. MTL DEVELOPMENT

A. Method of Approach

It is known that the efficiency of an electrically small loop antenna generally increases as the cube of the radius of the loop. Efficiency increases with loop radius (or frequency) until it reaches an asymptotic limit determined by other factors which are described in Section IIID. As this relationship reduces the uncertainties of size and scaling, an arbitrary fixed loop size was chosen for all tests, based on data taken at VHF, for most of the measurements described in this report.

In order to optimize the efficiency of a loop to be contained in the 2' x 2' x 1' volume selected it was necessary to develop methods for measuring changes in antenna efficiency. Two methods (the Q method, and the Wheeler method) were used and are described in Section IIIC.

The configuration of the loop within this 2' x 2' x 1' volume is constrained by three factors. The most important factor is the need to

maximize loop radius. Two options are possible but to maintain the lowest profile (with vertical polarization) a rectangular loop 2' x 1' was chosen. The second factor, which determines the longest conductor length required for the lowest desired frequency, is the maximum to minimum capacity ratio of the loop resonating (tuning) capacitor. This factor will be discussed in detail in Section IIIA. The number of turns in the loop was then fixed at the number required to produce the conductor length needed (6 to 8 turns depending on the tuning capacitor used). The third factor is conductor size and shape. Increasing the diameter of round conductor beyond a certain point is self limiting due to the proximity effect discussed in Section IIID.

By these and other processes of determination and elimination the preliminary MTL test antenna was constructed of 2 1/2" wide copper strap wound on a 2' x 2' x 10" styrofoam core. The 8 turns were interconnected by removable copper links such that they could be connected in series for a total length of about 45 feet, or in parallel for a total length of 5 1/2 feet or several series-parallel combination lengths in between (see Figs. 1a and 1b). This coil was then supported on a 2" thick foam spacer above a 26" x 27" aluminum plate counterpoise.

B. Initial MTL Measurements

The initial tests with the strap MTL were in determining suitable combinations of strap interconnections and values of tuning capacitor C_A and matching capacitor C_B . The functions of these capacitors and the operational characteristics of a typical fixed length loop are described in Section IIIA.

Standard variable vane broadcast band tuning capacitors (≈ 400 pf to ≈ 15 pf) were used for this tuning range test which resulted in four overlapping bands:

Band 1:	1.8 to 4.5 MHz	8 loops in series
Band 2:	3.6 to 8.3	4 pairs in series
Band 3:	6.2 to 14.8	2 quads in series
Band 4:	9.3 to 30.5	8 loops in parallel.

[Note: To date no combination of unused turns has been found to be more efficient than to connect them in parallel with used turns.]

With the approximate required capacitance range determined, vacuum variable transmitting type capacitors were ordered (500 pf to 5 pf, 5 kv) and installed. The resultant tuning range and measured efficiency for the 4 bands are shown in Fig. 2. The Q factor, $f_0/\Delta f$ where Δf is the bandwidth between the half power points (VSWR = 5.8), for these four bands is plotted in Fig. 3. The relationship of Q to efficiency will be discussed in Section IIIC.

It can be seen from either Fig. 2 or 3 that the tuning ranges of Band 1 and Band 4 nearly overlap. If only seven turns were used for

Band 1 its tuning range, presently 1.7 to 5.7 MHz, would shift upward to approximately 2 to 7.5 MHz. The frequency range of Band 4 is presently 8 to 37 MHz. A larger tuning capacitor (1000 pf max.) would reduce the low end frequency to comfortably overlap the high end of band 1. Therefore it is possible with available capacitors to cover the 2-30 MHz range with only two loop configurations. The advantages and disadvantages of intermediate bands are discussed in Section IIIH.

C. Final Model Design

A tubular 6 turn MTL was constructed from standard 2 inch copper tubing and fittings (see Fig. 4). This model serves several purposes:

- 1) Comparison of efficiency between strap and tubular conductors
- 2) Permit efficiency and Q measurements on the "best" no switch low band loop configuration
- 3) Permit a demonstration of the capability of high power operation over the entire low band (to 1 kW)
- 4) Improve ease of operation and ruggedize

The characteristics of this tubular model are as follows.:

Tuning range: 2 to 10 MHz

Efficiency range: 1% to 30%

Bandwidth: 3 kHz minimum (3 dB)

Impedance over tuning range: Adjustable to exactly 50

Input power: 1 kW

Size: 26" x 27" x 14" (coil = 26" x 26" x 12")

Weight: 50 lbs.

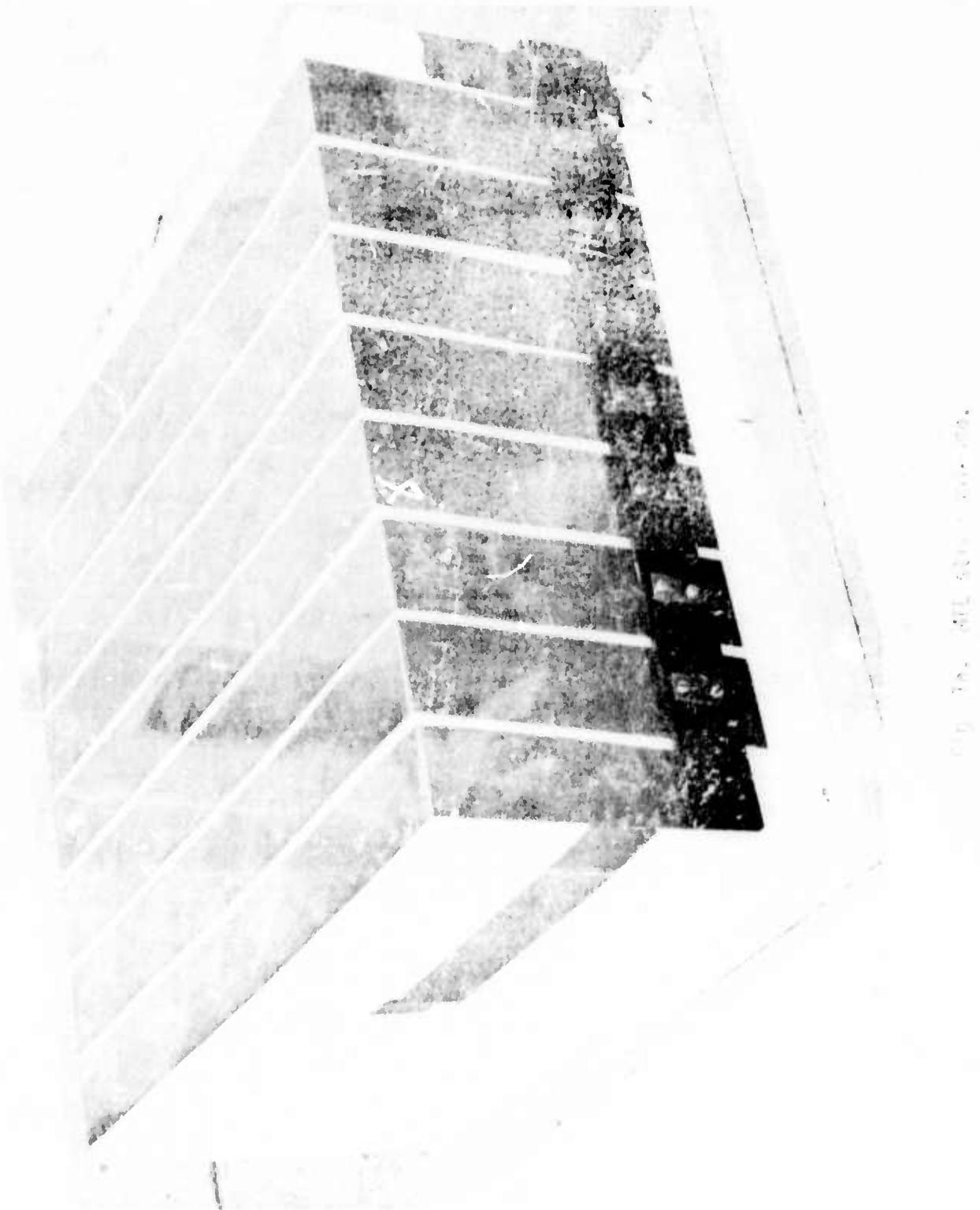
Calculated efficiency is plotted in Fig. 5 along with a single experimentally measured point at 10 MHz. With appropriate modification to connect each of the six turns in parallel (High Band) each specification would remain as before except

Tuning range: 10 to 32 MHz

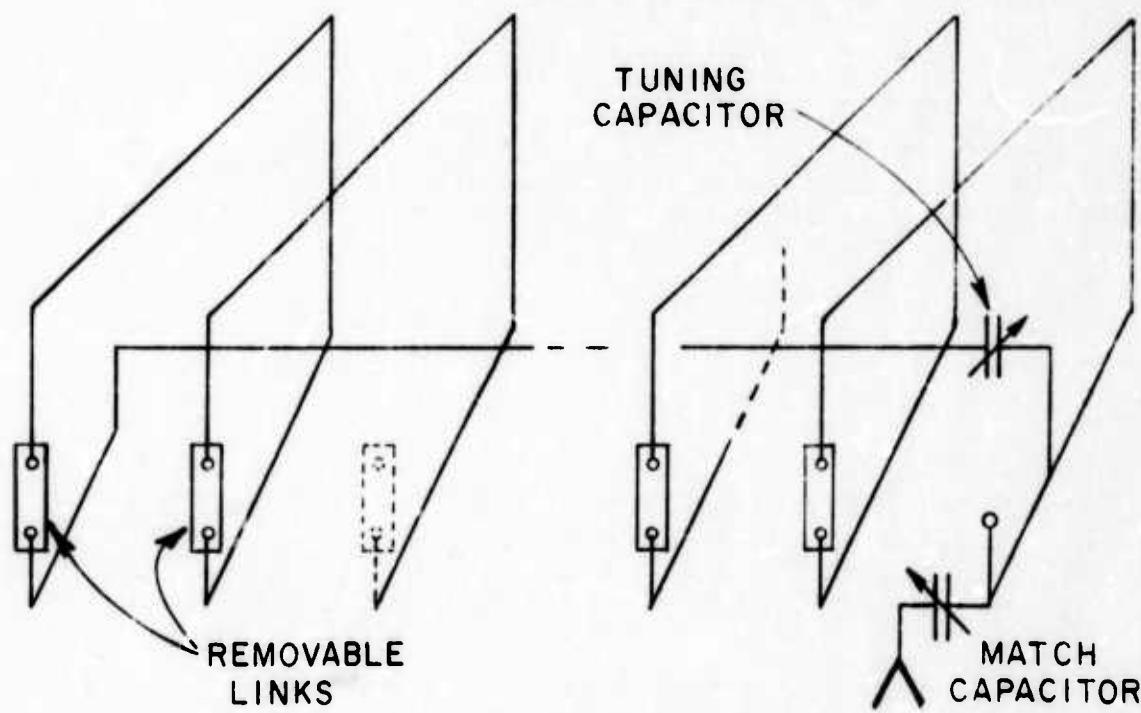
Efficiency: 10 to 80%

Bandwidth: 8 kHz minimum.

The switching method used with the strap MTL has been satisfactory for research purposes. It is felt, however, that additional work to improve and refine the method of switching between the low and high bands would be desirable. The problems associated with a single band, wide range, loop will be discussed in Section IIIH.



SERIES CONNECTION



PARALLEL CONNECTION

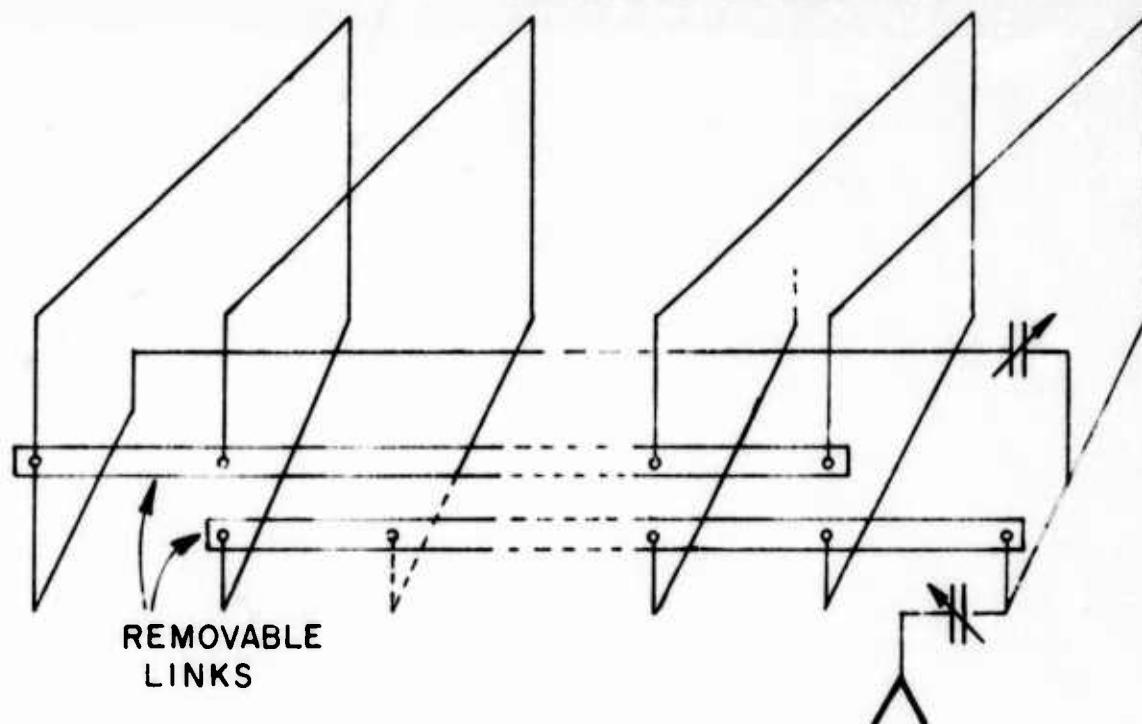


Fig. 1b. MTL strap antenna band switching technique.

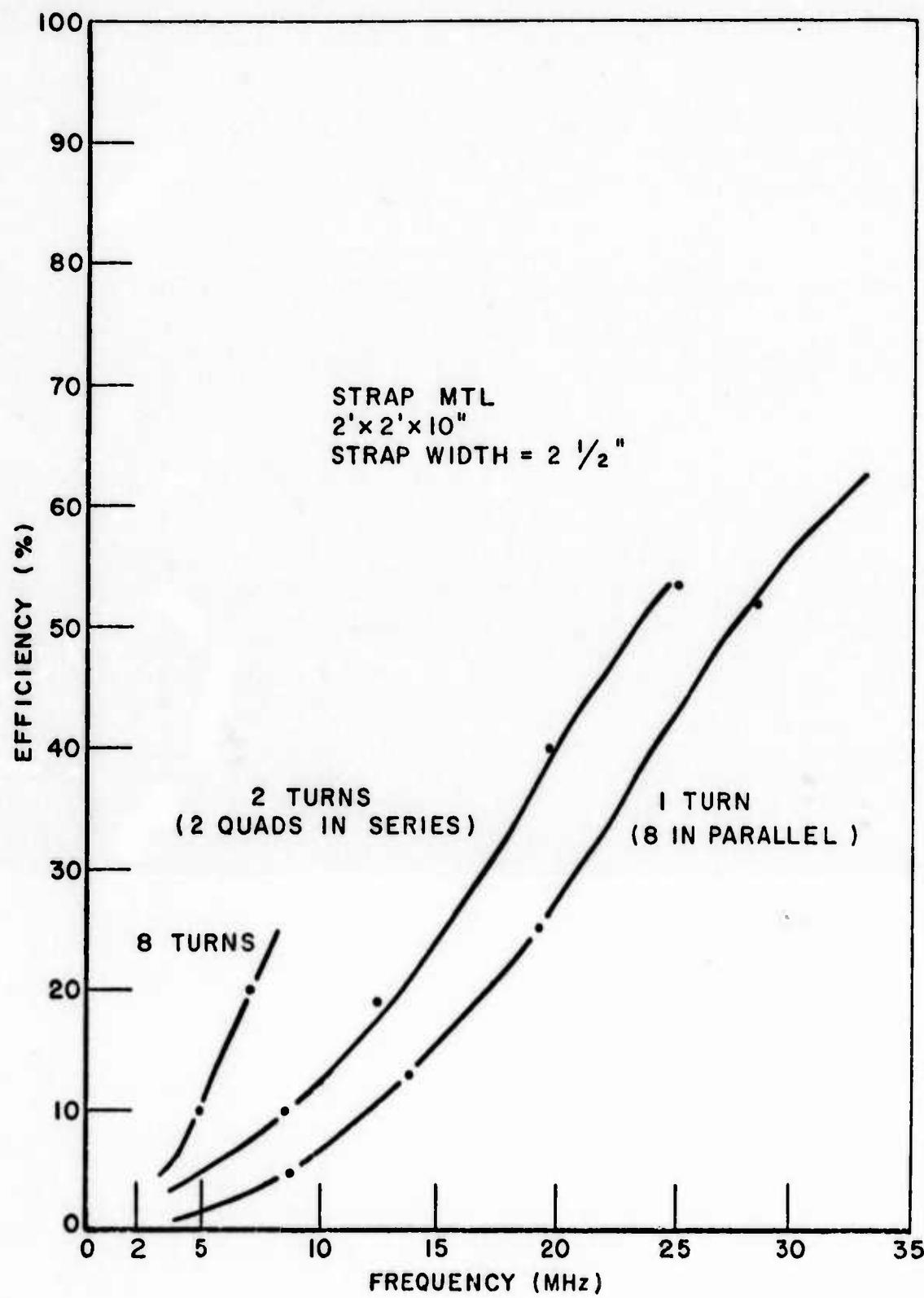


Fig. 2. MTL strap antenna efficiency.

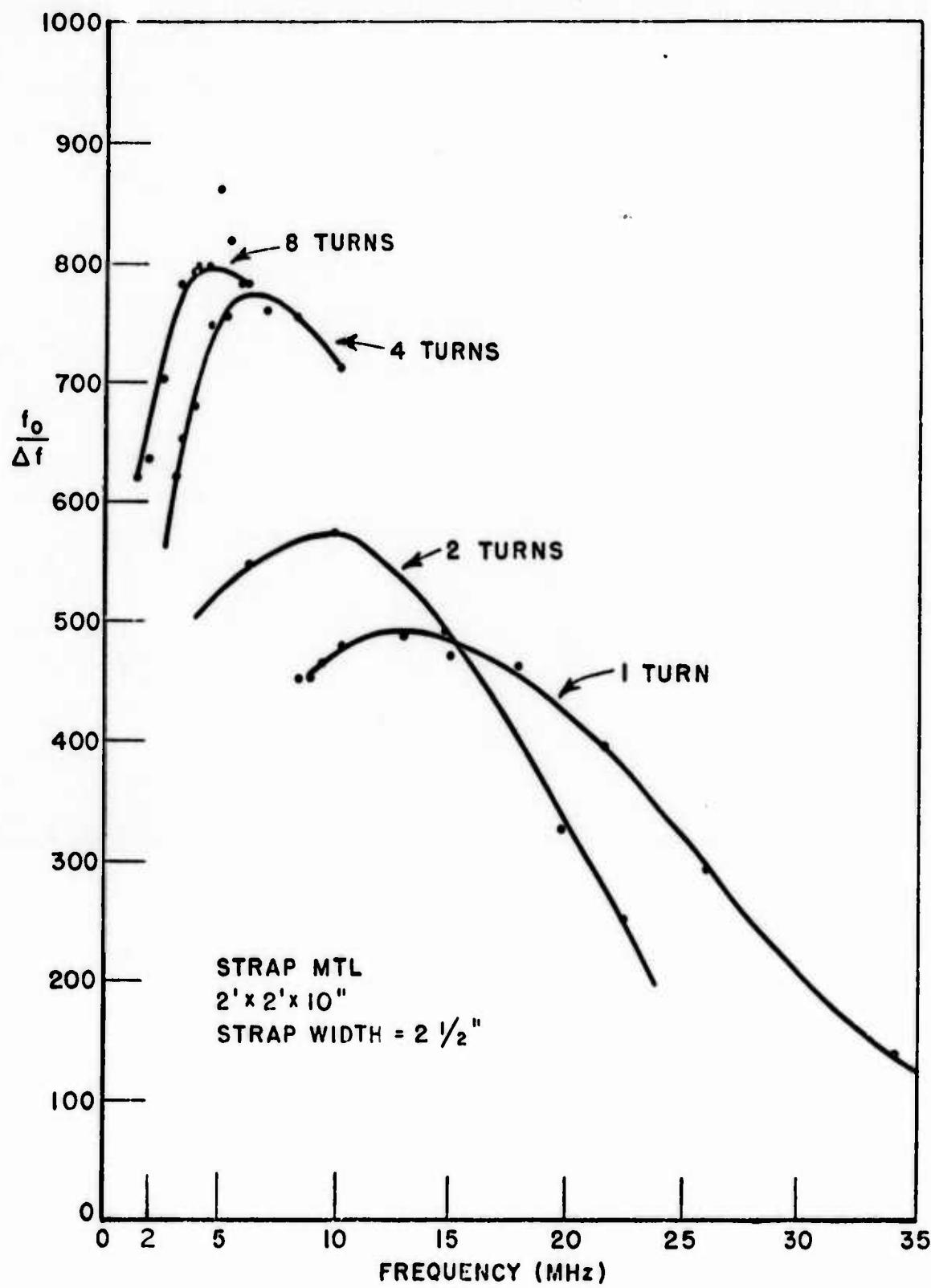


Fig. 3. MTL strap antenna Q factor.



Fig. 4. Tubular MTL antenna.

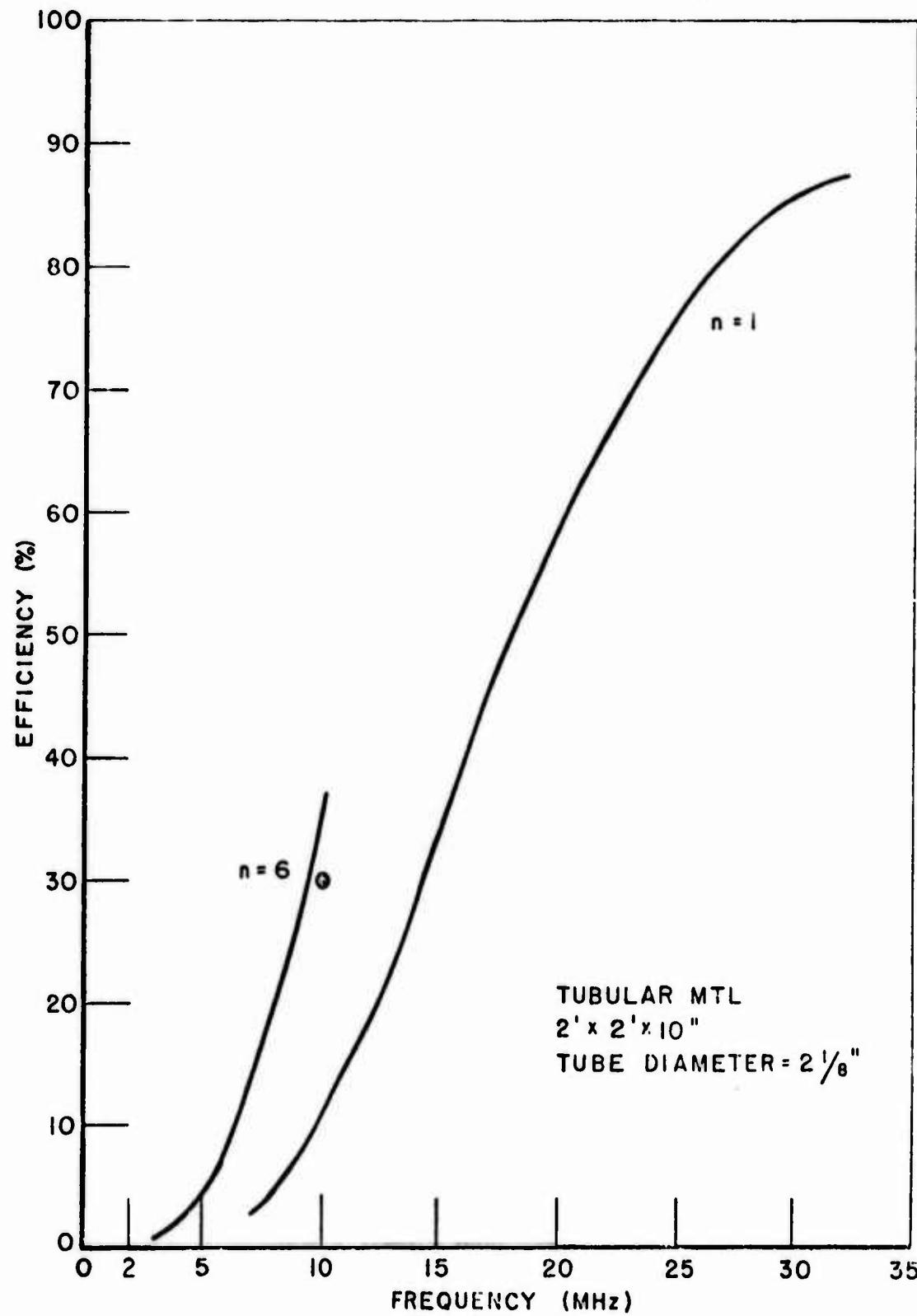


Fig. 5. Tubular MTL antenna calculated efficiency.

D. Size Scaling

It must be kept in mind that the design size of the prototype models was set so that efficiency at 3 MHz would be of the order of a few percent. This allowed greatest sensitivity to methods for optimizing efficiency for this particular size. A much larger size, with much higher efficiency would have made it more difficult to isolate and measure the effects of individual parameters. As will be shown in detail in a later section, doubling the loop diameter will result in an approximate efficiency increase of 8 when the efficiency is very low. The theoretical model described in Section IIID when standardized by the efficiency measurements made on the 2' x 2' x 1' model predicts a minimum efficiency of 6% at 3 MHz for a 2' x 2' x 2' model (Fig. 6). The required efficiency to size relationship may be adjusted to suit the needs of the problem and is not limited to the efficiency achieved in the 2' x 2' x 1' prototype.

III. MTL CHARACTERISTICS

A. Fundamental Resonance

The schematic representation of the basic multiturn loop antenna configuration is shown in Fig. 7. C_A (tuning capacitor) tunes the radiating element. The resultant impedance of this combination, shown in Fig. 8, is matched to the antenna feed cable by the match capacitor C_B . This configuration results in operation of the loop where the effective electrical length of the loop is always less than approximately $\lambda/2$. The loop input impedance is always inductive which permits the use of a nearly lossless capacitive impedance matching element.

The highest fundamental frequency of operation for a given loop is limited to the self resonant frequency of the loop. The actual frequency of operation (F_0) is slightly below the resonant frequency (F_r) as shown in Fig. 8. Additional parallel capacitance reduces F_r as defined by $F_r = 1/2\pi\sqrt{LC}$. The tuning band is, in practice, limited by the capacitance ratio C_{max}/C_{min} of available high power low loss capacitors. Typical maximum capacitance ratios of approximately 150 are available which would theoretically permit a 12 to 1 frequency ratio. In practice, about an 8 to 1 frequency ratio for a single turn loop and slightly less for a multiturn loop is the best that can be achieved due primarily to the fixed self capacitance of the loop.

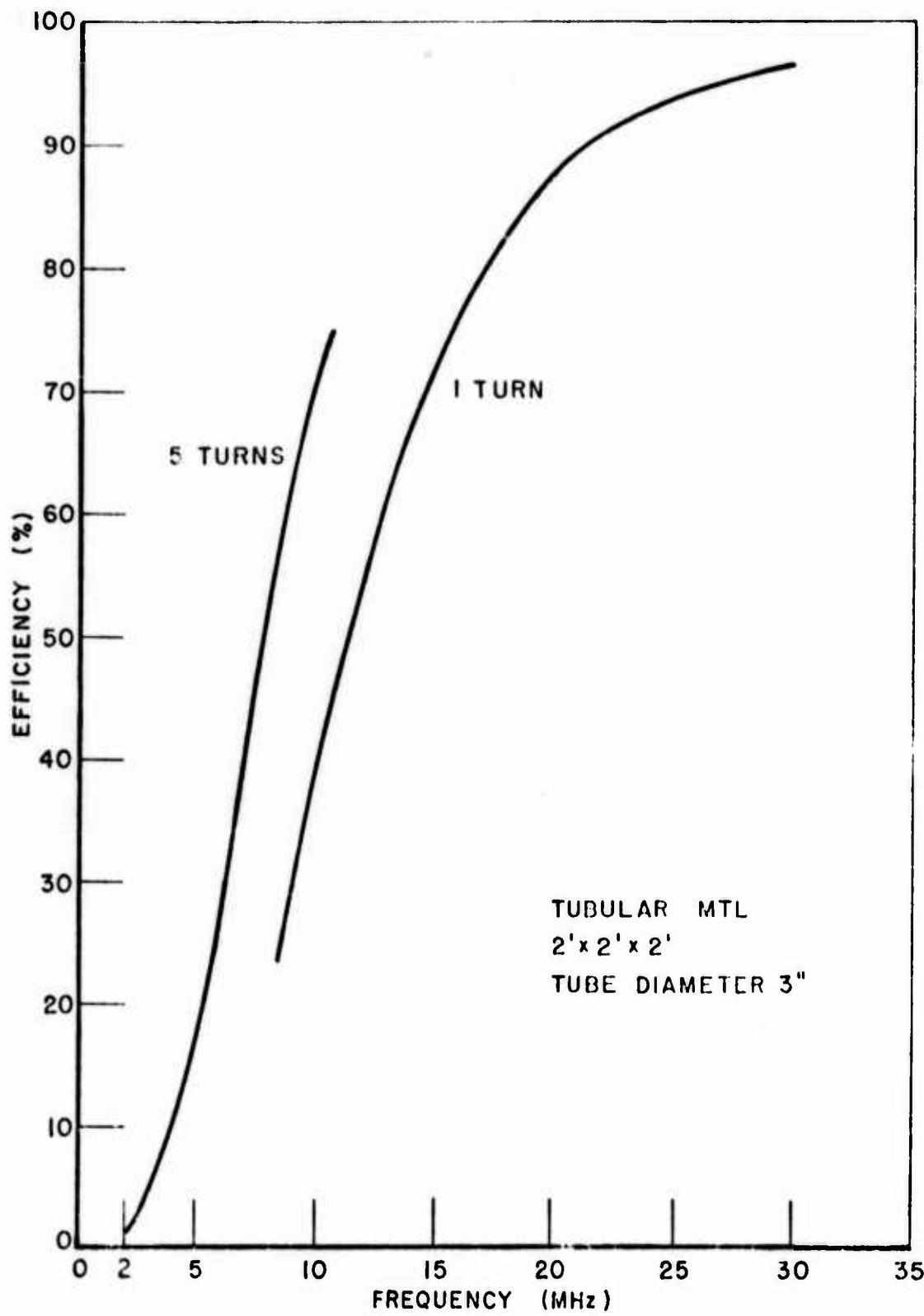


Fig. 6. Calculated efficiency of a $2' \times 2' \times 2'$ MTL antenna.

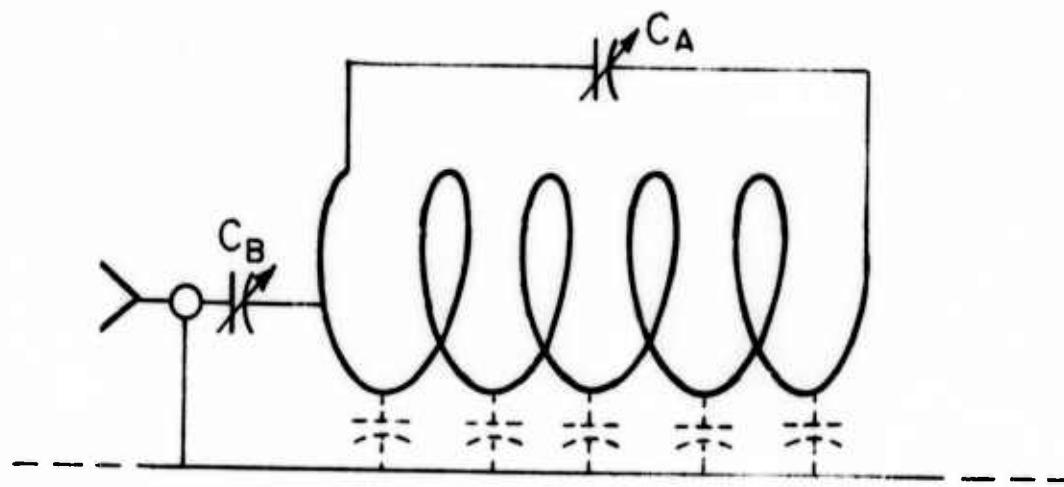


Fig. 7. Schematic diagram of MTL antenna.

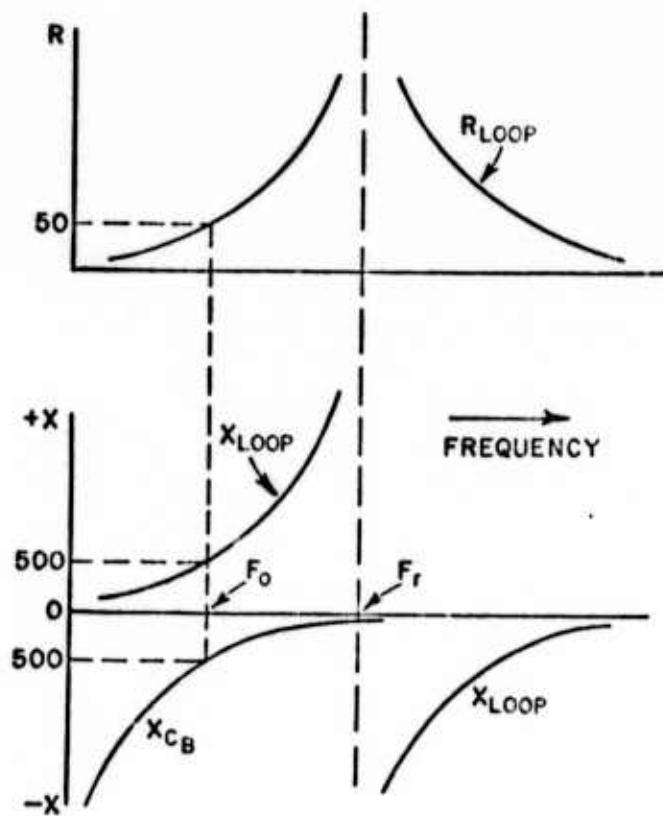


Fig. 8. Typical loop characteristic impedance.

B. Higher Order Resonances

As shown in Fig. 9, higher order resonances (f_3, f_5, f_7) exist at uniform spacings for a given coil. It can be seen that loop impedance conditions similar to those at f_1 exist at $f_3, f_5, f_7 \dots$. By reducing the size of C_B , impedance matching may be achieved at f_{03}, f_{05}, f_{07} , etc. In practice f_{03}, f_{05}, f_{07} , exist at slightly less than the odd multiples expected. Wire lengths, then, become about 0.8 to 1.2 for the band at f_{03} , and 1.5 to 1.7 lambda for the band at f_{05} . One obvious limitation to operation with a wire length of N (where N is any integer) is that the current distribution on the wire (normally wound coil) is such that the fields generated tend to cancel each other with no resultant radiated energy. Measurements of a VHF loop over several frequency bands from $\lambda/8$ to $5\lambda/2$ confirm that at λ and 2λ , efficiency drops to a very low level. Clearly, some reduction (ranging from mild to severe) in efficiency is present when a loop is operated at frequencies where loop length is greater than $\lambda/2$.

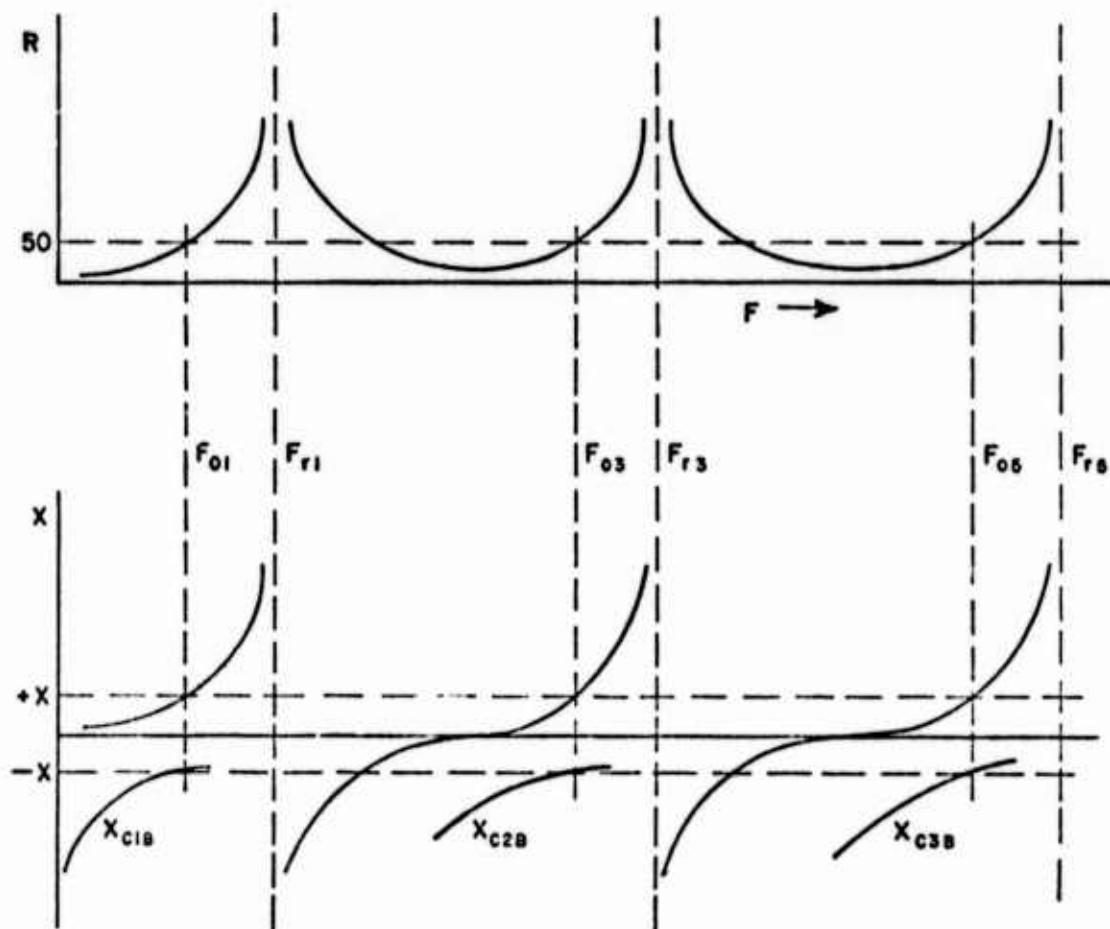


Fig. 9. Typical loop impedance match points.

The final 6-turn tubular MTL exhibits a f03 band of from approximately 16 to 24 MHz. The minimum loop efficiency of this band is unknown but is expected to be low due to self cancellation at a frequency where loop length is one wavelength. Due to limited time no tests were made to confirm this expectation. It is recommended that these tests be carried out when the Navy evaluates this antenna.

C. MTL Efficiency

In order to evaluate the effect on efficiency of loop parameter changes it is necessary to be able to measure small changes in efficiency accurately. The two methods chosen for this purpose were the Q factor method and the Wheeler method.

The Wheeler method, which has proven very effective and reliable for MTL work at VHF, was suggested by H.A. Wheeler in a paper published in 1959 (Ref. 1).

The efficiency of an antenna may be found from the relation

$$E = \frac{R_r}{R_L + R_r}$$

where R_r = radiation resistance

R_L = loss resistance.

The real portion ($R_L + R_r$) of the input impedance is easily determined. R_L (or R_r) is more difficult to determine accurately. Wheeler suggests that a conducting sphere a radian length (one-sixth wavelength) in radius or larger surrounding the antenna will eliminate R_r without significantly changing R_L . This assumes no change in current distribution on the antenna. The measured input resistance will, with these assumptions, be R_L . Both of these measurements are easily accomplished with a network analyzer such as the H.P. 8407 or G.R. 1710. In practice $R_L + R_r$ is determined from the analyzer Smith chart when the antenna sees free space and R_L when the sphere is in place around the antenna. Further details of the development and limitations of this method may be found in Ref. 2.

It was determined from previous work (reported in Ref. 2) that the shape of the "conducting sphere" was not critical. For H.F. measurements a 15' x 15' x 10' chamber was constructed of aluminum tubing and bronze screening. See Fig. 10. The Q factor of this chamber at a natural resonance frequency of about 45 MHz was approximately 6000.

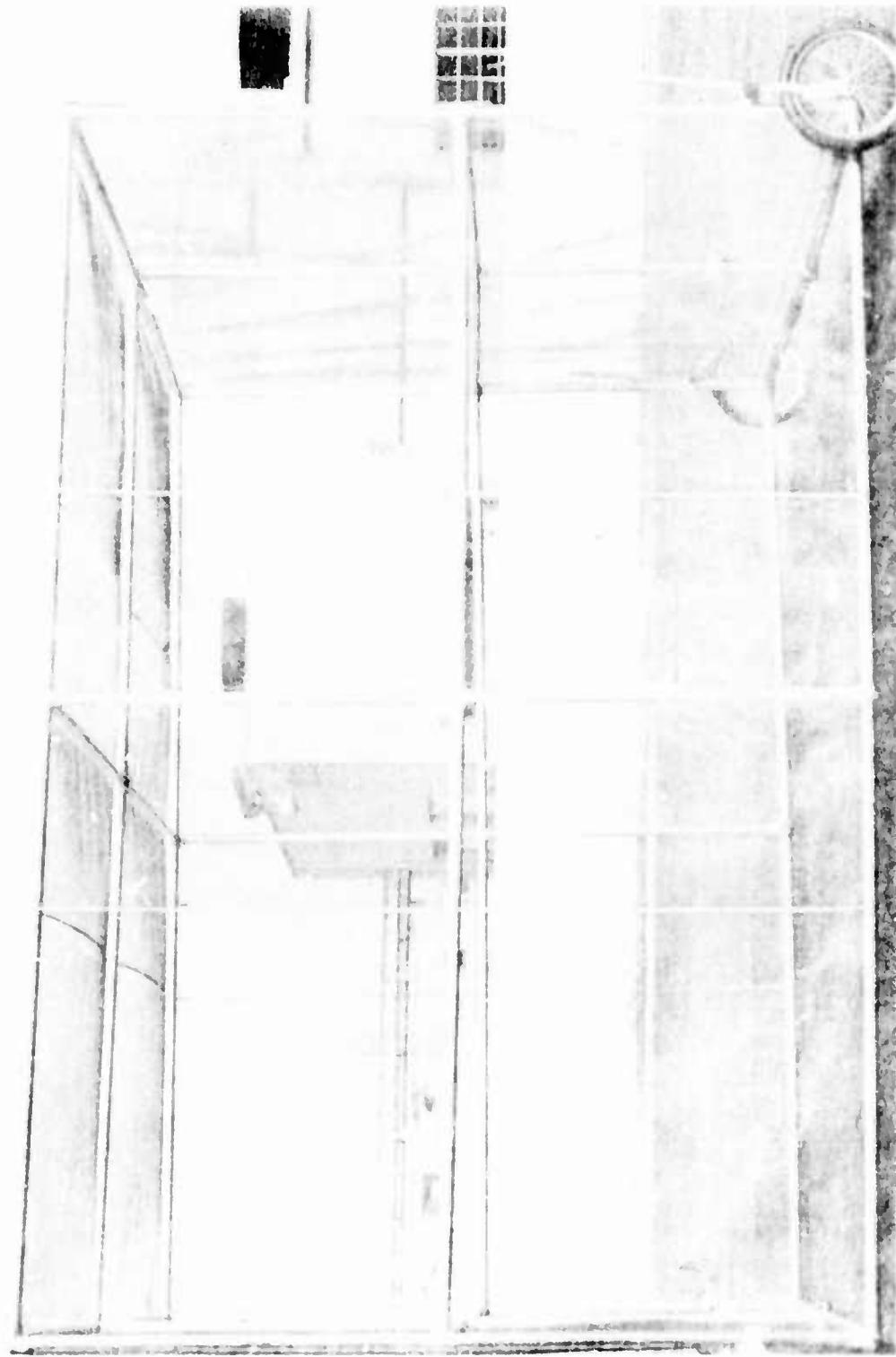


Fig. 10. Wheeler method conducting cap and ground plane.

The Q factor method of efficiency is based on a comparison of measured to ideal Q. Q is defined as

$$Q = \frac{\omega \times \text{peak energy stored}}{\text{average power dissipated}}$$

If Q is high it can be interpreted as the reciprocal of the bandwidth (Δf) of the antenna times the operational frequency of the antenna where Δf is defined as the bandwidth between the frequencies for which the resistance is equal to the reactance. This corresponds to the frequency at which the power absorbed by the tuned circuit is one half the amount absorbed at resonance. In the case of an antenna matched to a transmission line the half power frequencies that determine Δf occur for a power reflection coefficient of 0.5 (SWR = 5.83).

If we have a lossless antenna (power dissipated only in radiation) we define Q as

$$Q_R = \frac{\omega \times \text{peak energy stored}}{\text{average power radiated}}$$

The Q of a realizable antenna is then defined

$$Q_{RL} = \frac{\omega \times \text{peak energy stored}}{\text{average power radiated} + \text{average power dissipated}}$$

If we have the same antenna structure in both cases and if the current distribution on the antenna is not severely altered by the losses, then the stored energies may be assumed to be equal and the antenna efficiency is simply the ratio

$$\eta = \frac{Q_{RL}}{Q_R} = \frac{\text{power radiated}}{\text{power radiated} + \text{power dissipated}}$$

Formulation of Q_R (1st order mode) is presented by Chu (Ref. 3) as a function of Ka (Fig. 11), where a is the radius of the smallest sphere into which the antenna will fit and $K = 2\pi/\lambda$. The calculation of antenna efficiency as shown by the solid curves in Fig. 15 was made in this manner with measured Q data from the actual antennas involved.

D. Comparison of Efficiency Data with a Theoretical Model

The recently available work of Smith (Ref. 4) is used in this section to enlarge upon and compare experimental data with a theoretically derived model. Smith's MTL antenna model is based on a lossless feed system, constant current over the length of the wire

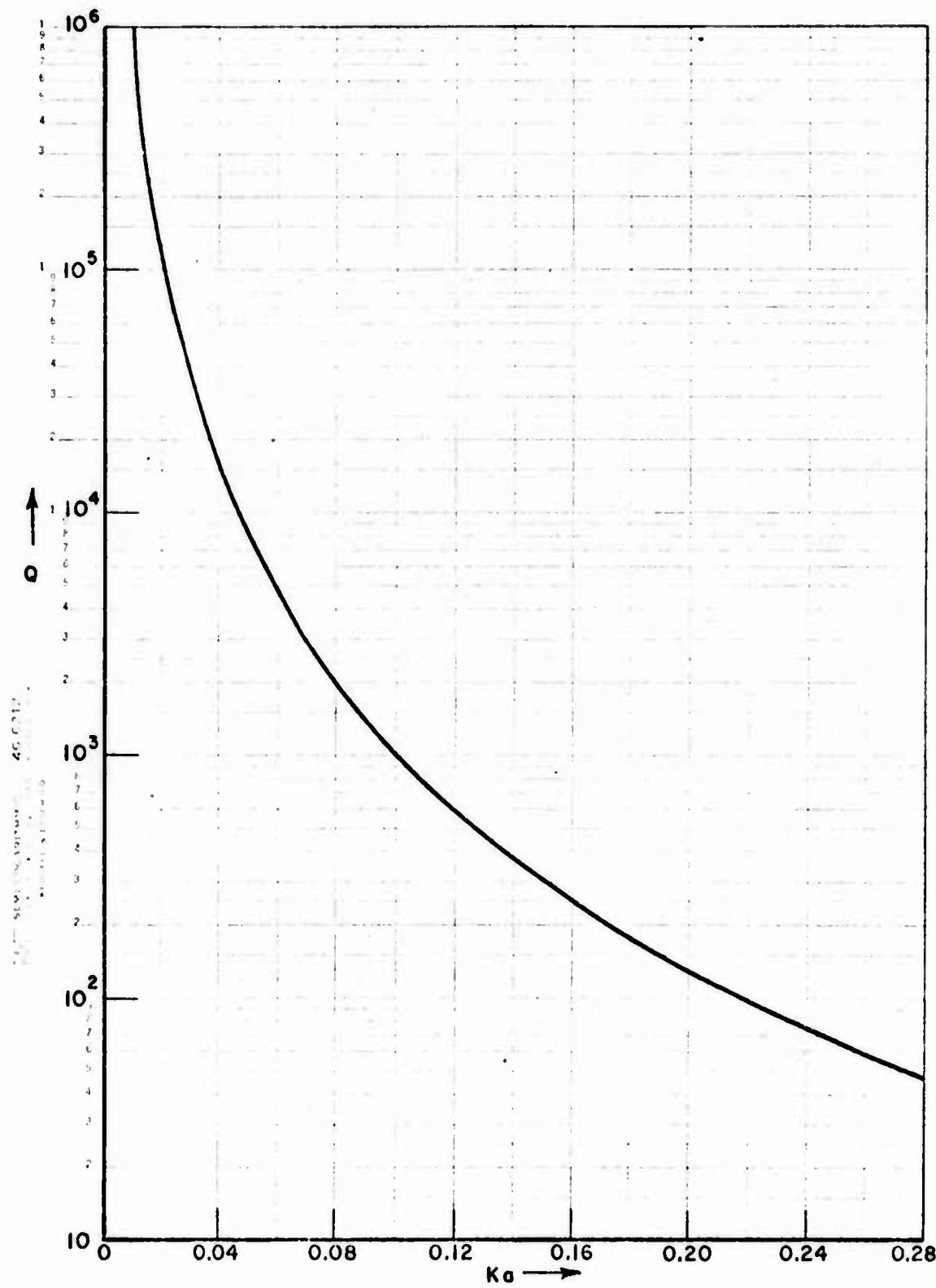


Fig. 11. Theoretical ideal Q as a function of K_a .

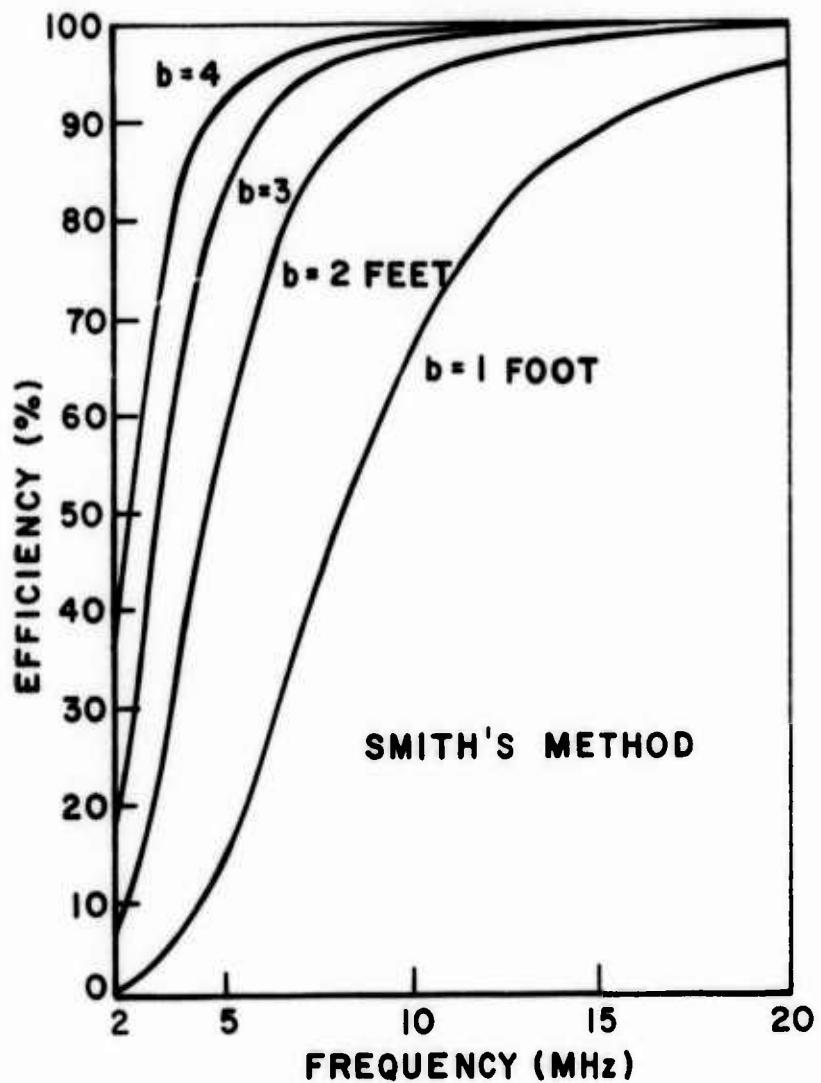


Fig. 12. Theoretical loop efficiency as a function of coil radius.

and a coil configuration where coil length is much smaller than coil diameter. Its most convenient form is written:

$$E_a = \frac{1}{1 + \frac{8.48 \times 10^{-10} \sqrt{F_{\text{MHz}} \sigma_r}}{n(b')^3 a'} \left(1 + \frac{R_p}{R_0}\right)}$$

Where E_a = Coil efficiency

F_{MHz} = Operational frequency in MHz

b	= Radius of loop
a	= Radius of wire
σ_r	= Ratio of conductivity of loop wire to that of copper
b'	= Radius of loop normalized to free space wavelength
a'	= Radius of wire normalized to free space wavelength
n	= Number of turns
R_o	= AC wire resistance per unit length
R_p	= AC wire resistance per unit length due to proximity.

As previously mentioned, the major factor controlling loop efficiency is loop radius. As the

$$\frac{8.48 \times 10^{-10} \sqrt{F_{\text{MHz}} \sigma_r}}{n(b')^3 a'} \left(1 + \frac{R_p}{R_o}\right)$$

factor begins to dominate the denominator, E_a varies directly as the cube of loop radius, linearly with wire diameter and inversely with wire proximity. The proximity effect, or increase in wire resistance due to current displacement in closely spaced wires, is discussed in detail in Ref. 4.

For our purposes its effect is to limit loop wire diameter for a given wire spacing. Of course, the R.F. resistance of an isolated wire is inversely proportional to its diameter (skin depth at H.F. is 1 to 2 mils). Two or more parallel current carrying wires whose axes are a fixed distance apart will influence each other. Smith shows that increasing the diameter of these wires will result in decreasing R.F. resistance only to the point where mutually induced currents begin to reduce the useable area of the wire. Beyond this point any further increase in wire diameter increases R.F. resistance. For a 6 turn loop the optimum ratio of half the wire spacing (c) to wire radius (a) recommended by Smith is approximately 1.5. The ratio c/a for the tubular model is 1.8/1.1 - 1.6.

Figure 12 illustrates the effect on efficiency of changing coil radius (b), holding the number of turns and wire size constant. Figure 13 shows changing efficiency with increasing turns, holding coil radius and wire size constant. Proximity loss is zero in both cases. Figure 14 indicates that a single turn loop requires a radius increase of $24/13 = 1.85$, to achieve the same efficiency as a 6 turn loop (neglecting proximity loss). Generally, as discussed in the next section, the rate of change of efficiency is probably somewhat more flat and does not decrease as fast at the low end, nor reach a high efficiency asymptotically as rapidly as the Smith equation suggests. It is felt, however, that the relative changes shown are helpful in visualizing trends and isolating first order parameter effects.

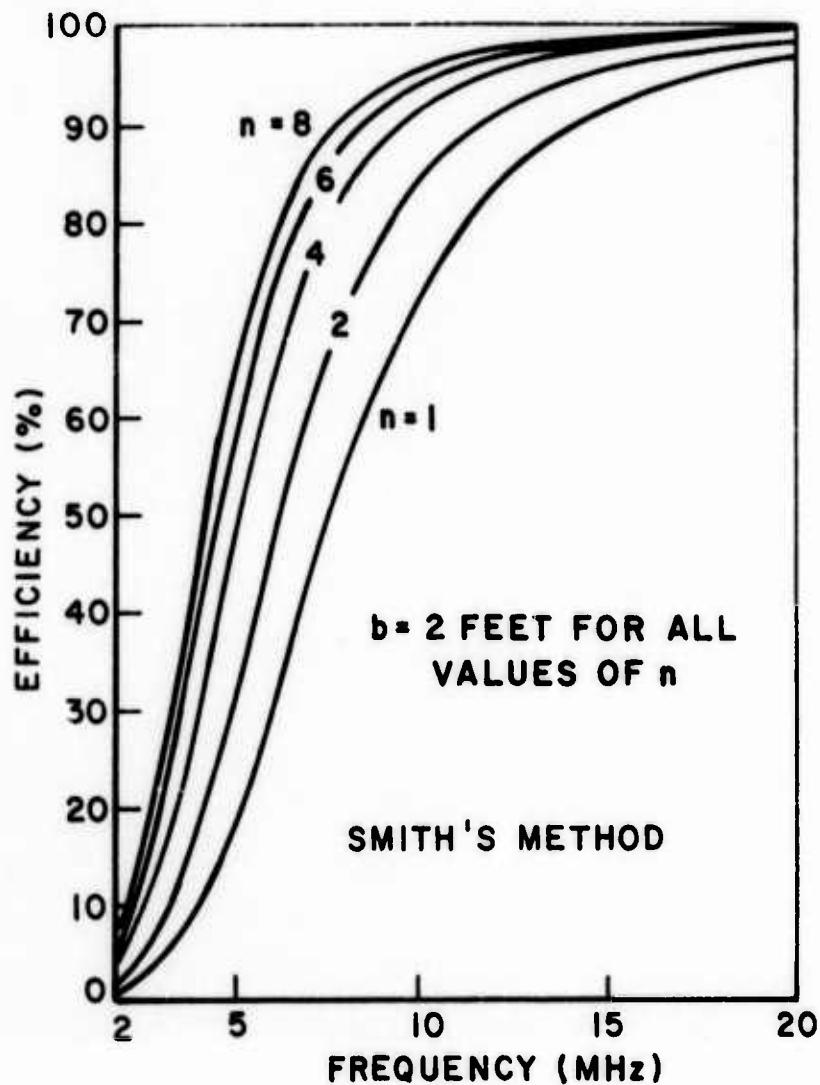


Fig. 13. Theoretical loop efficiency as a function of coil turns.

E. Comparison of Efficiency Measurement Methods

Perhaps the most interesting feature of the efficiency measuring techniques represented in Fig. 15 is the rather good agreement of the three methods for the 8 turn coil. For this particular antenna and frequency range the ideal Q used in the Q factor method is changing rapidly - (145,000 at 2 MHz, 9000 at 5 MHz) while measured Q is reasonably constant between 650 and 800. The Wheeler method in this range is becoming limited due to large reactive shifts when the cap is in place, reducing the reading accuracy of the resistive shift.

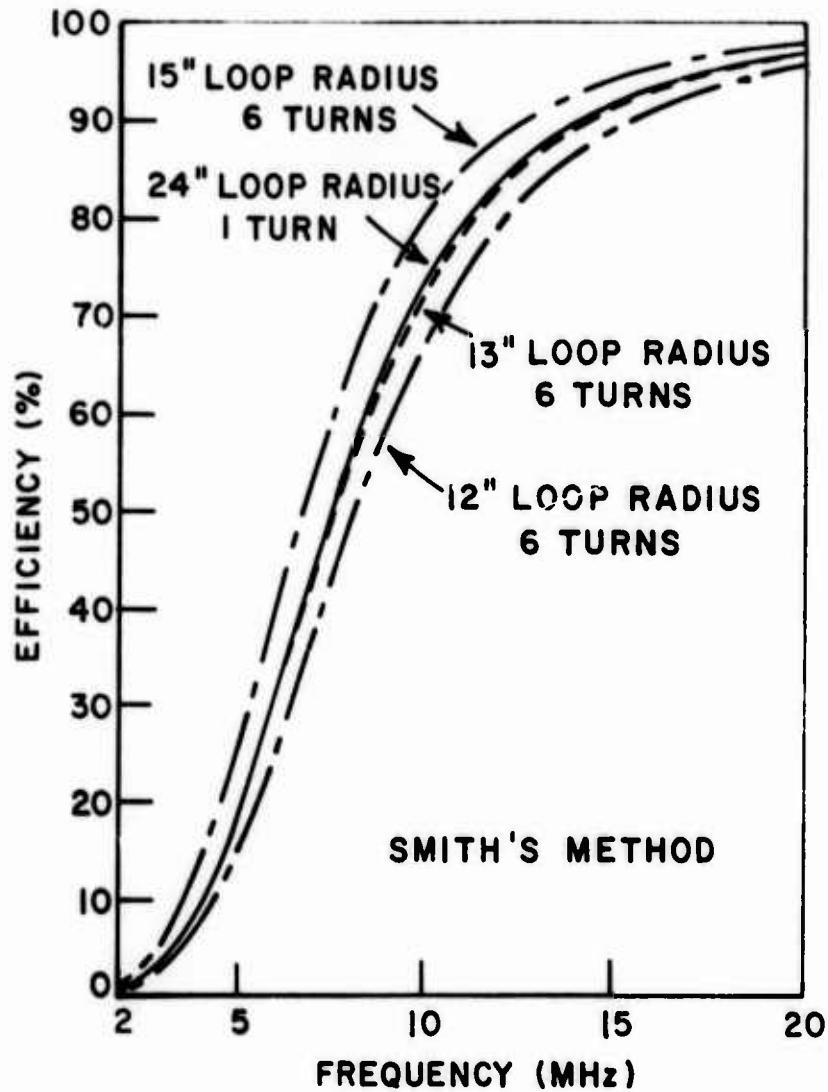


Fig. 14. Sizes and number of turns.

Smith's method, which is based on a coil of round wire (arbitrary correction factor of 1/2 for strap wire) in free space, is in error because of loop coupling to the ground environment at the lower frequencies. The maximum disagreement between the three methods, for the 8 turn coil is less than 3 dB which, considering the limitations of the methods is rather good.

The rather large spread of results for the one turn loop is due principally to limitations in the Q factor method. The value of Q_{used} for the one turn loop was reduced slightly to reflect the probability that only the 1st turn is radiating significantly. The Wheeler method results differ by less than 3 dB from those calculated by Smith's method. One possible explanation for the increasing Q factor differential at higher

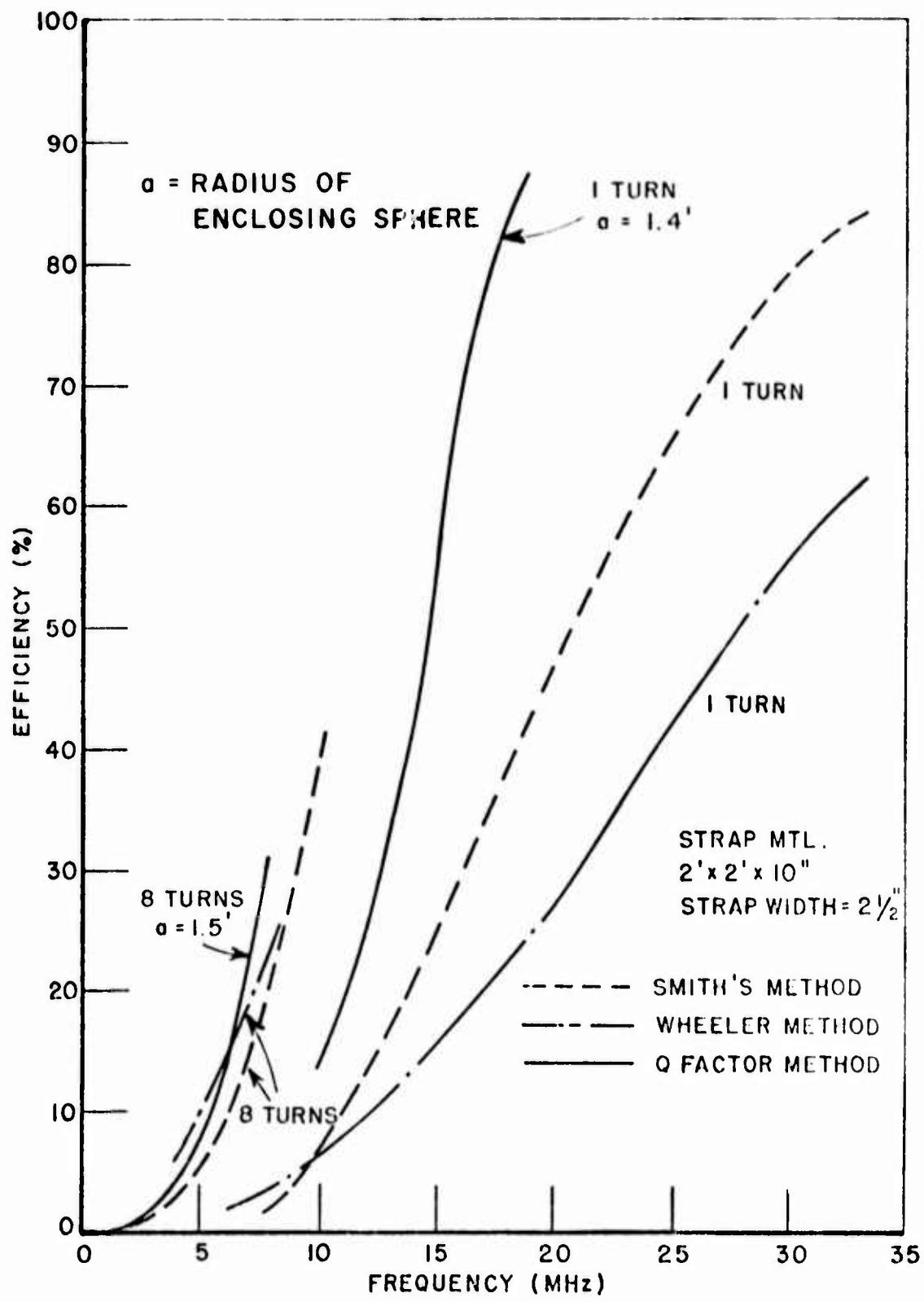


Fig. 15. Comparison of MTL efficiency determined by three different methods.

frequencies is a basic limitation of the method. Briefly, the method is based on the number of spherical modes of radiation possible for a given radiator. Ideal Q increases with the number of possible modes. If we calculate ideal Q based on the assumption that only the 1st order spherical mode is present when in fact some 2nd and 3rd modes are present, the ideal Q will be less than the model requires for reasonable accuracy. As $n = \text{Real } Q/\text{Ideal } Q$, n will be greater than would be predicted with proper use of the model. The presence of these higher order modes would be difficult to determine quantitatively but is suggested by the slight narrowing of the H-plane patterns at the higher frequencies. Even though the percentage of these higher order modes is small, and would be expected to be small for antennas whose major dimension are never greater than $\lambda/15$, their effect on accurate determination of antenna efficiency can be appreciable. Further information on this subject can be found in Ref. 3.

One consistent trend for both 8 turn and 1 turn loops is that the Wheeler method results in a slightly lower rate of efficiency change with frequency. At this point, based on experience at VHF as well as H.F., greatest reliance is placed on the Wheeler method for determining antenna efficiency.

With the present 15' x 15' x 10' cap, reasonable accuracy can be obtained down to 10 MHz. Below 10 MHz, accuracy is dependent on efficiency level, improving with decreasing efficiency. A correction factor due to the finite Q of the cavity used in the Wheeler method to surround the antenna has not been applied to any of the curves shown. This correction factor would increase the early results by about 10 per cent and the later results by about 20 percent. The difference reflects the decreasing Q of the cavity with time.

F. Cavity Recession

Figure 16 shows the effect of recessing the strap MTL into a cavity. Clearly, there is a significant reduction of Q factor and thus efficiency over part of the band. The smaller cavity (30" square) results in an efficiency reduction of up to 3 dB while the larger cavity (36" square) causes less severe reductions especially at the high end of the band. When cavity recession is essential, cavity dimensions should be as large and as shallow as possible to maximize loop efficiency. Standard radome techniques are acceptable and should add no appreciable loss.

It is strongly recommended that where possible, surface or bulkhead MTL mounting with radome cover for protection be used. While these test results are limited, similar results were observed during earlier work at VHF which substantiates the probability that MTL antennas mounted in small cavities are almost always significantly less efficient than when mounted above the ground plane.

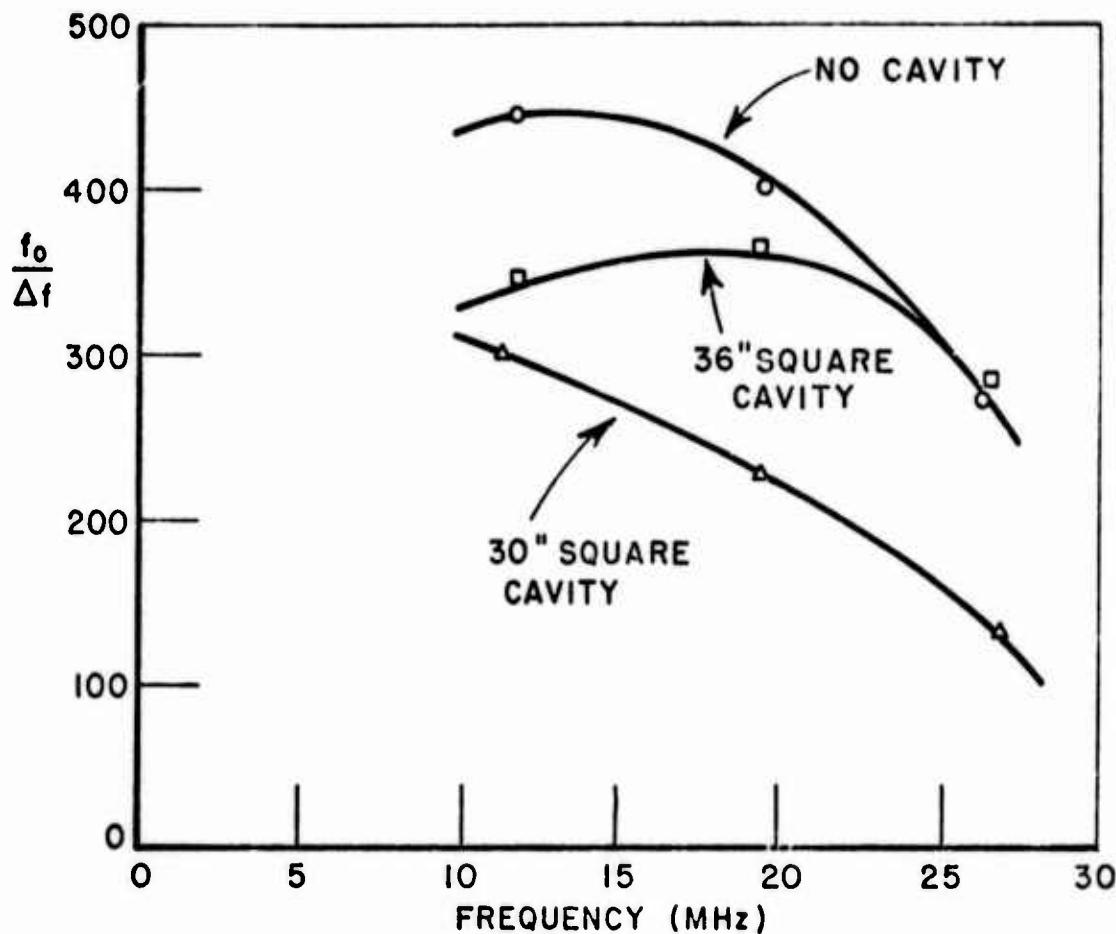


Fig. 16. Measured Q as a function of cavity size.

6. Ground Plane Effects

The MTL configuration used for this work uses no direct coil to ground plane connection. The feed line return path is through the distributed capacity between the coil and ground. It was found that for several frequencies measured that a voltage null exists near the center of the loop. This null shifts somewhat as a function of frequency, but it may be possible to ground this center point if desired without significantly reducing loop efficiency as has been done with VHF MTL's. The significance of this null may be more easily understood if the coil is visualized as an artificial transmission line, with the individual turns forming the lumped inductive elements and the capacitance between each turn and the ground plane forming the capacitative elements. The tuning capacitor must be replaced by two capacitors in series whose combined value is equal to that of the capacitor actually used (Fig. 17). The left hand capacitor in the figure has the value necessary to resonate that portion of the line to the left of the voltage minimum. If the

**TUNING
CAPACITOR
IN TWO PARTS**

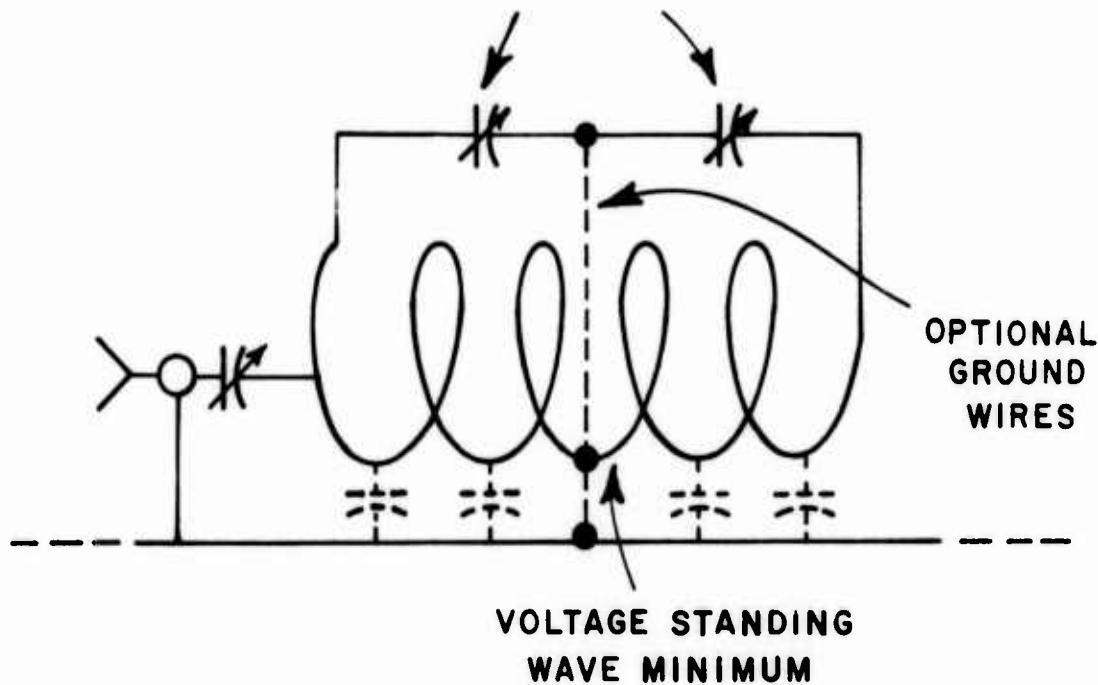


Fig. 17. Representative MTL schematic.

voltage minimum point on the coil and the midpoint between the two capacitors are now grounded the circuit is not appreciably changed. The left hand end simply becomes a secondary circuit energized by the primary winding on the right. The right hand end of the coil then appears as a length of lossy transmission line shorted at the far end. The loss is the combined effect of its own resistive loss, and radiation resistance, plus the coupled loss and radiation resistance from the other end of the coil. The right hand half of the tuning capacitor and the input capacitor may then be thought of as a conventional series parallel capacitive matching network, such as is used to match any arbitrary load to a transmission line (50 ohms). It should be noted that it is desirable to have a voltage minimum near the center of the coil since this is also a current maximum and maximum current means maximum radiation.

The characteristics of the grounded loop have not been studied in detail. Additional time should be spent on this aspect of loop operation to provide additional insight into MTL behavior.

The size of the ground plane used for all measurements described was 15 x 15 feet. In one test this ground plane was extended to 23 x 23 feet without noticeably affecting tuning, impedance or efficiency characteristics at 6, 13, 29 MHz. At 3 MHz the Q factor increased by 5 (698 to 733) when the 23 ft square ground plane was used.

The height of the loop above the ground plane was changed to determine the importance of this factor. The three height settings (measured from the bottom of the coil to the aluminum plate) were 2, 4, and 6 inches.

Frequency in MHz	2"	4"	6"
27.4	248	257	256
13.3	456	470	487
6.0	722	740	738
3.0	680	744	768

While some improvement is seen (about 10%) at 3 MHz it appears that small adjustments to the design height are of limited value.

The nominal value of Q factor for the low band was found to vary from day to day. Principal causes were moisture content of the grass surrounding the ground plane and the number of grass blades protruding through the fine mesh bronze screen ground plane used. A polyethylene sheet was used under most of the ground plant to reduce this variation.

Ground plane conductivity is an important factor in MTL efficiency. Further work needs to be done in this area in consideration of the effects of a shipboard environment. A reduction in Q factor of the strap MTL over a period of several months was found to be caused by increased resistivity in the ground plane. This ground plane, as well as the entire conducting cover used for the Wheeler method efficiency measurements is made of a fine bronze wire mesh. Q factor measurements taken shortly after assembly of this equipment showed a cavity Q factor of 6200. Tests made 3 months later showed a cavity Q factor of 3500.

Surface weathering, particularly in the wire to wire junctions of the mesh, are felt to be principal contributors in reducing the cavity Q. Suitable adjustments can be made to determine actual values possible with an adequate ground plane. However, the ground plane for an operational antenna needs careful consideration to avoid needless loss in antenna efficiency.

Although it is not clear at this point what the exact size of ground plane should be, it does appear that copper sheet should be used under the antenna and in the immediate vicinity. The Q method mentioned previously would provide a simple test for the size ground plane needed in an actual installation. That is, if the antenna is known to have a Q of about 1000, for example, with a satisfactory ground plane, then one

could add sufficient copper sheeting at the actual installation to achieve a similar Q. It could be assumed that the efficiency of the antenna would be altered by the same factor as Q.

H. The Single Turn Loop Antenna

Consider the loop efficiency equation

$$E_a = \frac{1}{4.48 \times 10^{-10} \cdot F_{MHz}^2} \cdot \left(1 + \frac{R_p}{R_0}\right) \cdot \frac{1}{1 + \frac{n(l')^3 a}{1}} \quad (1)$$

whose parameters are defined in Section IIID. As previously shown adding seven turns to a single turn loop at a given frequency can increase loop efficiency as much as doubling the single turn loop radius. These seven turns of course, must be spaced far enough apart to avoid significant (R_p/R_0) loss due to proximity effect. An advantage of increasing loop efficiency by adding turns rather than increasing loop radius is that adding turns increases loop size in one dimension rather than two as for the increased radius loop.

As previously mentioned the frequency range over which a loop may be tuned is limited by the capacity ratio of the tuning capacitor. The best ratio achieved to date by a commercial antenna for a single turn loop is approximately 8 to 1.* In practice the multturn loop tuning range (without band switching) is slightly less than this due to increased inter-wire capacity. The dual band model described previously is capable of tuning over a 15 to 1 range.

When antenna requirements call for the smallest element possible for a specified low band efficiency the multturn loop antenna has a decided advantage. If the radius of a single turn loop is increased so that the efficiency at the edge of the low band (f_L) is equal to the MTL (at f_L), mid band efficiency will be somewhat greater for the single turn loop but will probably not exceed the MTL by more than 3 dB. Some improvement in the midband efficiency of the MTL can be made, when band switching techniques are used, by interconnecting turns for the longest loop possible that still operates in the fundamental resonance mode. For a fixed loop diameter this results in the maximum number of turns in the loop and thus greatest efficiency

*Antenna Research Associates MLA-2

I. High Power Measurements

The strap MTL (with styrofoam core) was tested at 4 frequencies (3.5, 7.1, 14.2, 29.5 MHz) at high power. Jennings model USCL-500-5S vacuum variable capacitors (5000 volts) were used for C_A and C_B . No noticeable breakdown occurred when, for short periods (≈ 10 seconds), up to 700-1000 watts were applied to this antenna at 29.5 MHz (High Band). The Q factor for this band never exceeds 500. When the same test was run on the low band, capacitor breakdown occurred in the tuning capacitor at 3.5 MHz at approximately 400-500 watts. Q factor for the low band is approximately 800 at 3.5 MHz. It was also noted at this time that for long periods of operation at a power level just below breakdown, the temperature rise of the tuning capacitor (surrounded by foam in the center of the coil) rose significantly. The thin strap used to connect this capacitor to exposed coil windings was not sufficient to keep the capacitor temperature at an acceptable level. Future designs should include means for good heat transfer away from this element as well as good ventilation for convective cooling.

In order to upgrade the final test model (6 turn tubular) for operation at high power levels, a larger, 15,000 volt test, tuning capacitor (Jennings UCSXF-1000-15S) was ordered and installed (Fig. 4).

In tests with the prototype strap model, no wire to wire breakdown problem was observed even though wire spacing was only 0.3 inch. Generally, it is not expected that any extraordinary measures are necessary in MTL design to avoid voltage breakdown.

J. Tuning and Tapping

Two basic MTL band switching methods are possible where no significant loop configuration changes are necessary. The least satisfactory of the two methods involves the use of the 3rd, 5th, 7th, etc. resonances of the fundamental loop. It's disadvantages lie in the "holes" or nonuseable bands between useable bands (i.e., with a 2-10 MHz fundamental band, the next useable bands are approximately 16-24 MHz and 34 to 42 MHz). The field distribution in the loop in these useable bands begins to be self cancelling (when coil length is greater than $\lambda/2$) which results in a reduction in loop efficiency when compared with a loop of the same diameter but fewer turns.

The second method starts with the strap MTL connected for low band operation (1.7-7 MHz). Seven of the 8 straps are shorted along one side of the loop. The resulting tuning range is from 8 to 29 MHz. Measured efficiency, however, is from 3 to 6 dB below the value achieved with all 8 straps in parallel (High Band).

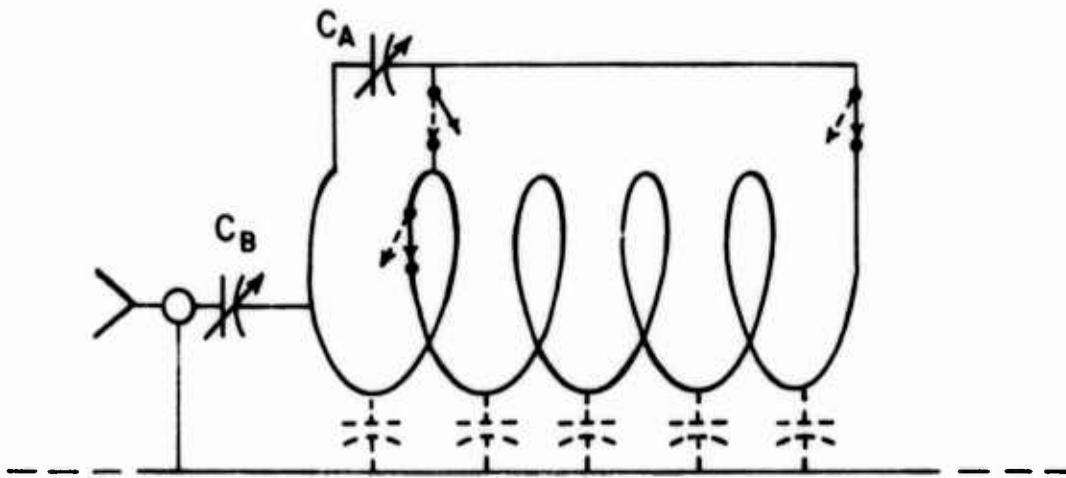


Fig. 18. An MTL band switching technique.

How then, can we operate an MTL most efficiently in the fundamental mode over a wide frequency range? Several methods were tried. Figure 18 shows, schematically, a capacitor switching method. The unused turns were, in turn, left connected to the active loop or disconnected electrically but left in place. The efficiency of this combination was always significantly less than if the unused turns were removed completely and in some cases, depending on whether the unused turns were open or short circuited, the tuning range was seriously limited. A possible advantage to this approach to band switching is that the switching might be done capacitatively thus avoiding switch contact loss problems. This approach has not been thoroughly exhausted and should be studied further.

The most efficient location for unused loop turns is in parallel with used turns. Slight increases in efficiency (over the case where the turns are physically removed) result due to an effective increase in loop wire diameter. Switch or pressure plate contact loss, normally a deterrent to good antenna design, has not proven troublesome in the 6 month test period of the strap MTL. Further study is needed into the limitations of this approach.

K. Simultaneous Transmit/Receive

At this point a simultaneous transmit/receive capability over the entire H.F. band does not seem feasible. Simultaneous resonances that are not harmonically related are difficult to produce and highly interactive. Multi-frequency operation with harmonically related resonances ($\times 3, \times 5, \times 7$) is possible, particularly in the receive modes, but the frequency bands possible are limited. As shown in an earlier section the band around f_{03} is the same width as the band at f_{01} and extends downward from about 3 times the maximum frequency in band f_{01} .

For the tubular model MTL described, this band is from 24 MHz down to about 16 MHz. The ratio of 3rd order to 1st order frequencies at which a reasonable impedance match can be achieved varied from 8 (16/2) to 2.4 (24/10). It will be necessary to accept some compromise in impedance matching capability for this mode of operation as the two frequencies require different match capacitor values.

A second, somewhat more flexible, method of simultaneous operation could be developed in the form of a variety of loop interconnections involving additional tuning and matching capacitors. Here various DC isolated LC combinations can be formed using the turns on the basic loop. The major problem with this approach would be the effects of mutual coupling between elements of different DC isolated circuits. These effects may complicate the tuning and matching process considerably.

L. Comparison of Flat versus Round Wire

Consider two wires, one round and one flat, whose major crosssectional dimensions are equal. It is generally accepted that the RF resistance per unit length of the strap is about twice that of the round wire. If no proximity loss is present in a round wire MTL and the round wire is replaced by a thin strap wire of width equal to the diameter of the round wire, the MTL efficiency will be reduced by a maximum factor of 2 (at very low efficiencies). For initial efficiencies of greater than 50%, this factor quickly approaches 1. Reference 5, which presents studies of proximity loss for flat conductors, indicates that for thin conductors placed edge to edge, proximity loss is almost negligible. For applications where space is at a premium the increase in proximity loss for closely spaced round wires may substantially reduce or even eliminate the difference in RF resistance (and thus in MTL efficiency) between the two shapes of conductors. This assumes that the major loss in the antenna is the conductor RF resistivity.

Two of the models built during this program (one strap, one tubular) are similar, although not identical, in shape, and may be studied for differences. Although coil diameters are identical (to wire center lines) making b' for each the same, the tubular MTL has 6 turns of 2 1/8" diameter conductor while the other MTL is made of 8 turns of 2 1/2" wide strap. Let A denote the factor

$$\frac{8.48 \times 10^{-10} \sqrt{F_{\text{MHz}} \sigma_r}}{n(b')^2 a'} \left(1 + \frac{R_p}{R_o}\right)$$

The factor is modified by 8/6 for turns, 2 1/2/2 1/8 for conductor width, 1/2 for the round to flat approximation and (from Ref 4) by 1.6 for proximity loss elimination. Thus

$$A_{\text{strap}} = \frac{A_{\text{tube}}}{\frac{8}{6} \times \frac{2\frac{1}{2}}{2\frac{1}{8}} \times \frac{1}{2} \times 1.6} = 0.8 A_{\text{tube}}$$

which implies that where A dominates the efficiency equation the strap model should be slightly more efficient (~ 20%) than the tubular model. Actual measurements have shown just the opposite. The tubular MTL is about 25% more efficient than the strap MTL (Q factor method).

Two sources for error are the shape factor and the predicted proximity loss factor, neither of which has been measured. If the shape factor of 1/2 is reduced to 1/3

$$A_{\text{strap}} = 1.2 A_{\text{tube}}$$

which more nearly represents measured data. The proximity factor described and measured by Smith[4] does not consider the proximity of a nearby ground plane which will influence the conductors near it (over nearly 1/4th of the total length). If this factor is decreased from 1.6 to 1.3 (shape factor equals 1/2)

$$A_{\text{strap}} = 1.0 \times A_{\text{tube}}$$

As it is likely that proximity loss is greater than predicted rather than less, the principal error in the calculation of A_{strap} from A_{tube} is felt to be due to the conductor shape factor. In using the equation for future comparisons of this type, a 1/3 shape factor is recommended.

For designs where A is small (b' dominates) the cross-sectional shape of the loop conductor becomes electrically less important. The decision, then, as to when to use flat, round or other shape conductor may be considered as a dependent design parameter. In general, flat conductor is recommended where loop height must be kept to a minimum and loop length is not restricted. Here, the increased resistivity of strap compared to round conductor can be minimized by using very wide strap. When the most important space limitation is total volume, round conductor is recommended.

IV. MATHEMATICAL MODELING OF HF ANTENNA ON SHIP

Antenna performance is influenced by the environment in which the antenna is located. This is particularly true for an HF antenna on a ship. The pattern of the antenna may be altered dramatically and significant changes in impedance and efficiency also may be noted when the antenna is mounted on a ship. These effects are dependent on antenna position and some locations may actually enhance certain desirable antenna characteristics.

The usual procedure in the past for determining acceptable or desirable antenna locations has been to use experimental models of the ship and antenna, typically a 48th scale model. This is a slow and costly procedure if one attempts a parametric study to optimize the antenna system in some way. There now exists an alternate method that is far less costly both in time and money. This is the use of a mathematical model utilizing a high speed digital computer.

Techniques are available for developing a highly accurate mathematical model of the ship for electromagnetic analysis. These consist of the moment method for frequencies where the ship is less than about a wavelength in extent (generally frequencies below 10 MHz) and a combination of moment method and the geometrical theory of diffraction (GTD) for frequencies where the structure is larger than about a wavelength in extent. The moment method utilizes a wire grid or patch model of the actual structure or parts of the structure and a solution is found for the current distribution on the antenna and structure from which impedance, patterns, and efficiency may be found. The GTD method attributes radiation to each significant discontinuity in the structure, such as edges, masts, etc., as well as considering specular reflection. A combination of the methods can handle a structure of almost arbitrary size and complexity. Details of the methods are given in References 6 and 7.

The scope of the present program did not permit an extension of these existing techniques to an arbitrary ship. Rather, time and funding permitted only the computation of azimuth patterns for the rather simple box-like ship shown in Fig. 19. This corresponds to an actual model that has been used in a measurement program at NELC. Figures 20, 21, 22, and 23 show computed and measured azimuth patterns for a twin-whip antenna mounted at 45° with respect to the deck. The measurements were made at NELC and the computations were made, using the moment method, on the present program. The effect on the azimuth pattern when the whip antenna is replaced by a small loop is shown in Fig. 24, 25, 26, and 27. The pattern of a small loop in free space is included for comparison. Note that the structure of the ship provides sufficient regions for vertical currents so that at lower frequencies the pattern with the loop on the ship is more nearly that of a vertical monopole than that of a loop. At the low end of the HF band and for the location considered, the loop gives better omnidirectional coverage than does the 45° whip antenna.

No attempt was made to determine absolute gain in the above computations. The actual loop would be the HF multturn loop as described in the previous sections. For computations, only a single loop was used. However, it has the same area as one turn of the HF multturn loop. Experience has shown that the patterns are essentially the same for small multturn loops and single turn loops of the same area. However, the efficiency of the multturn loop is much greater than that of the single turn loop and hence the gain. One can get an estimate of the gain of the loop on the ship by estimating the directivity of the patterns in Figs. 24-27 and multiplying by the efficiency of the HF multturn loop taken from Figs. 2, 5, 6.

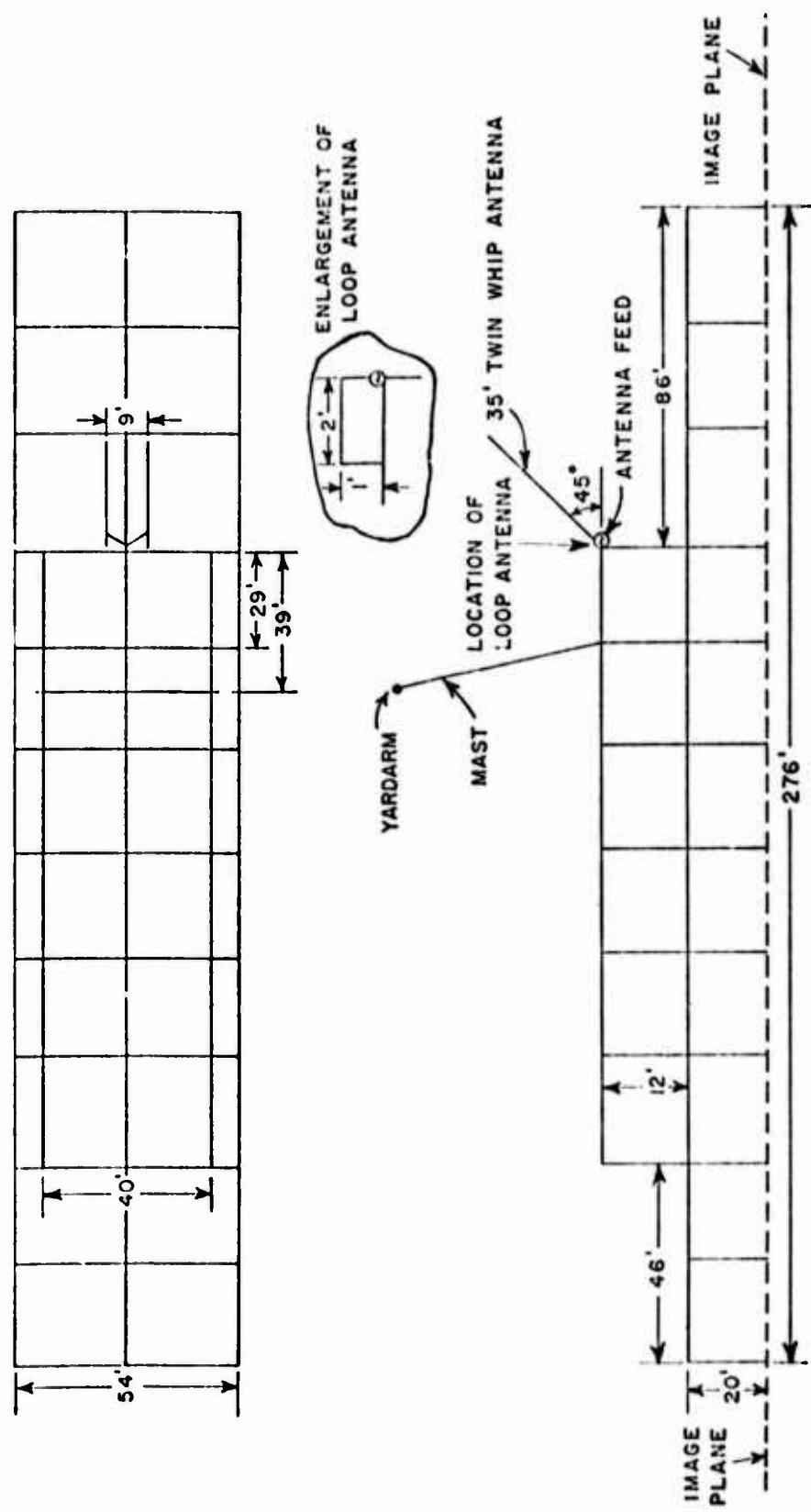


Fig. 19. Elementary ship model.

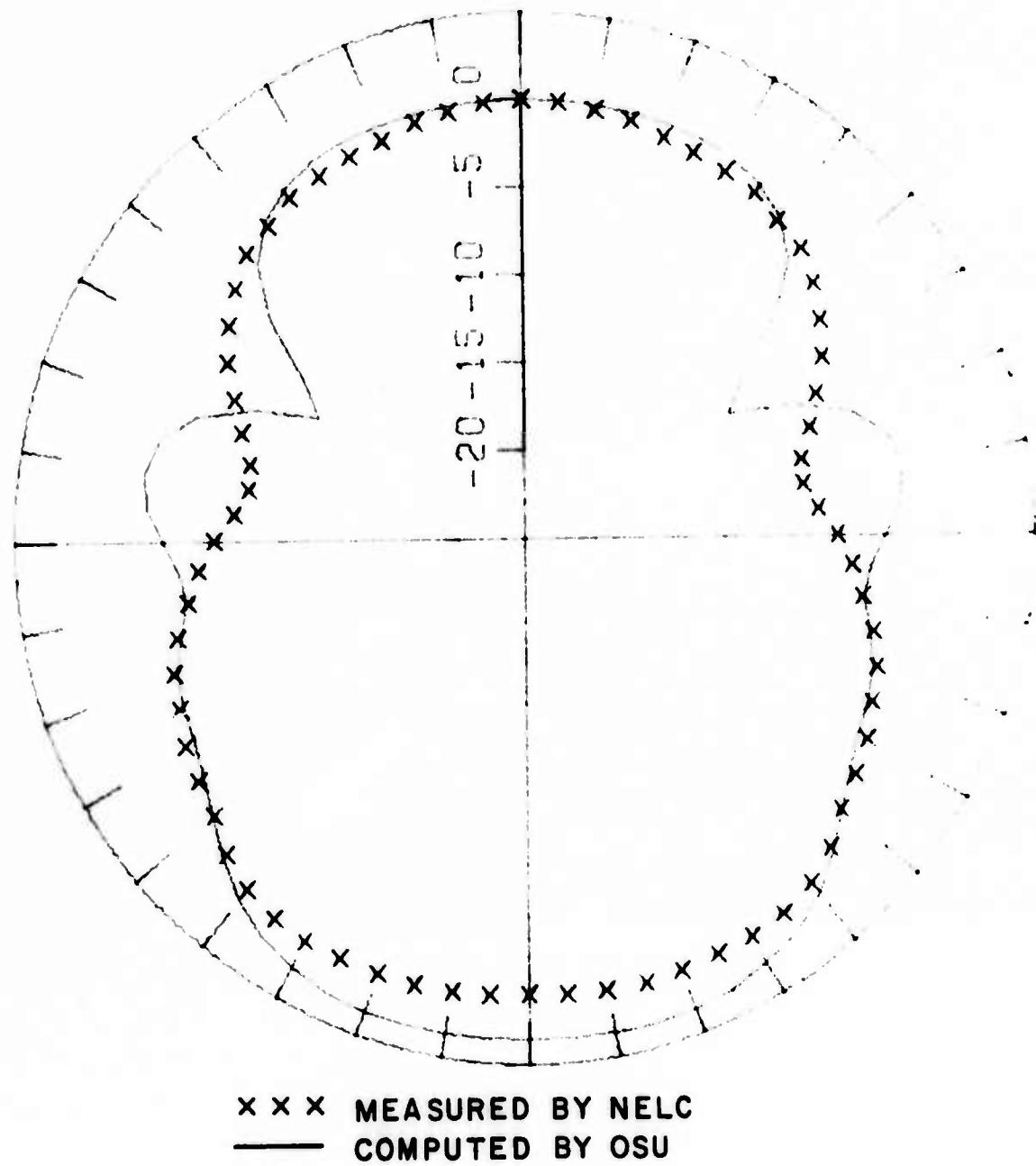


Fig. 20. Twin whip antenna pattern at 7.0 MHz ($\theta=60^\circ$).

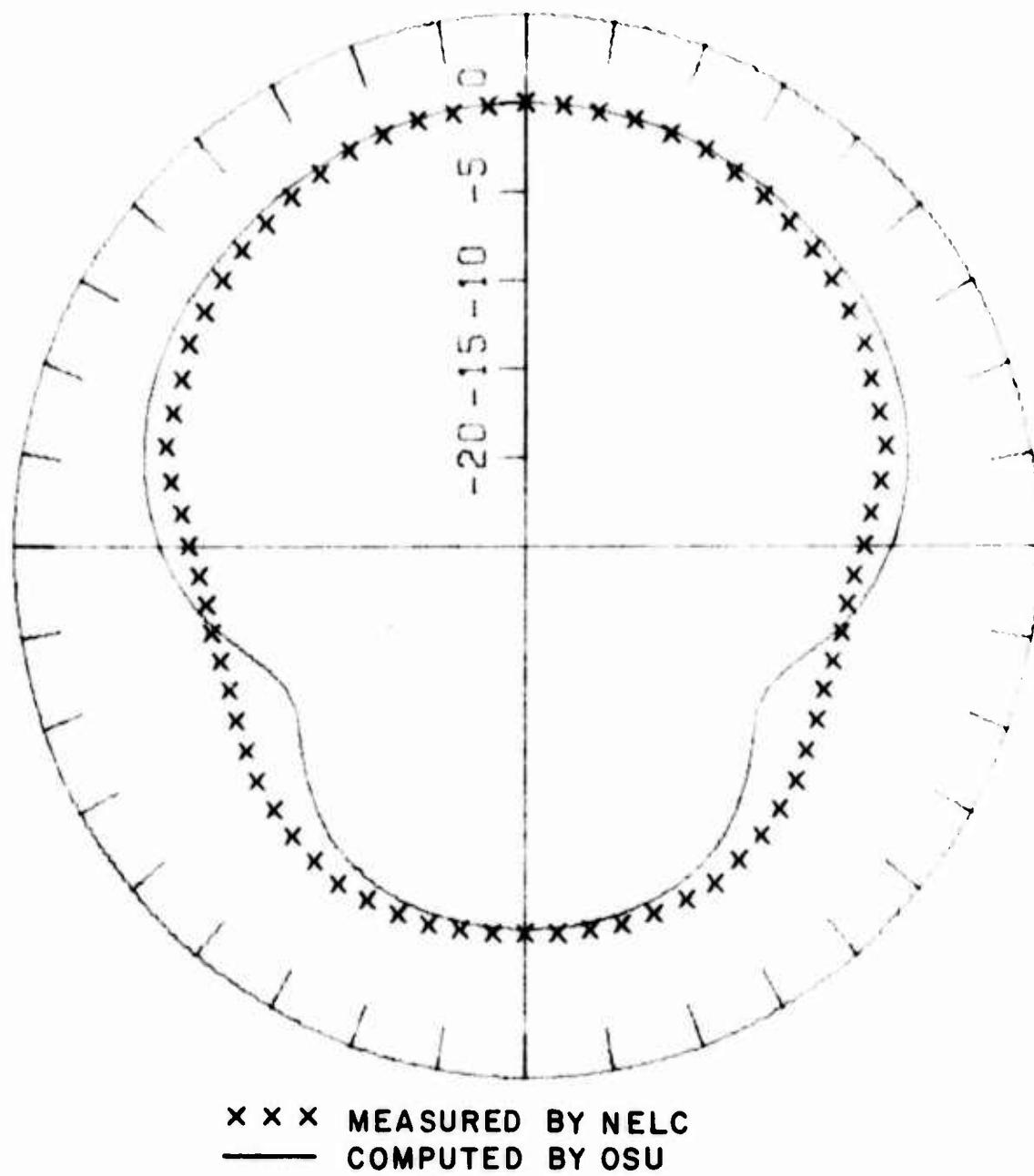
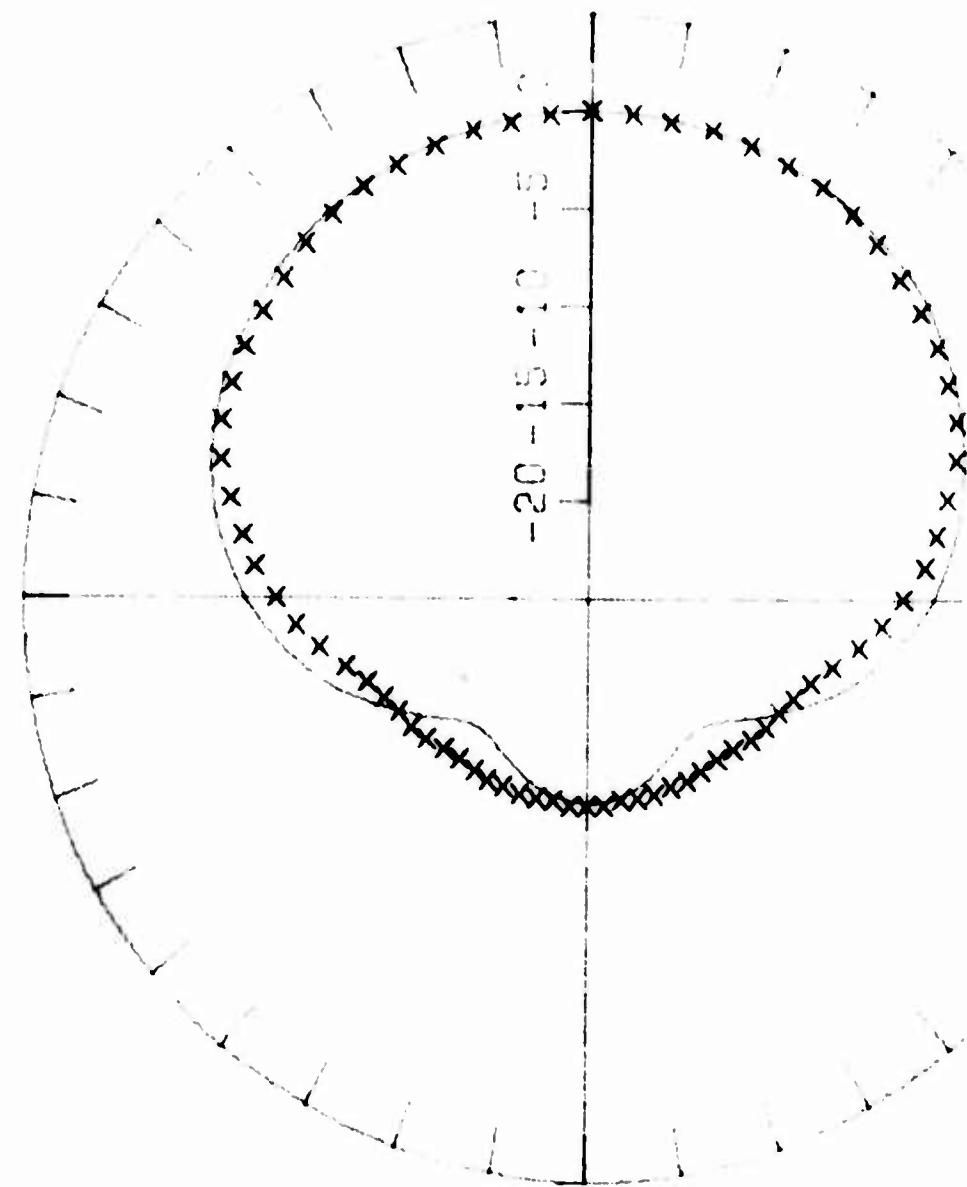


Fig. 21. Twin whip antenna pattern at 5.8 MHz ($\theta=60^\circ$).



× × × MEASURED BY NELC
— COMPUTED BY OSU

Fig. 22. Twin whip antenna pattern at 4.6 MHz ($\theta=85^\circ$).

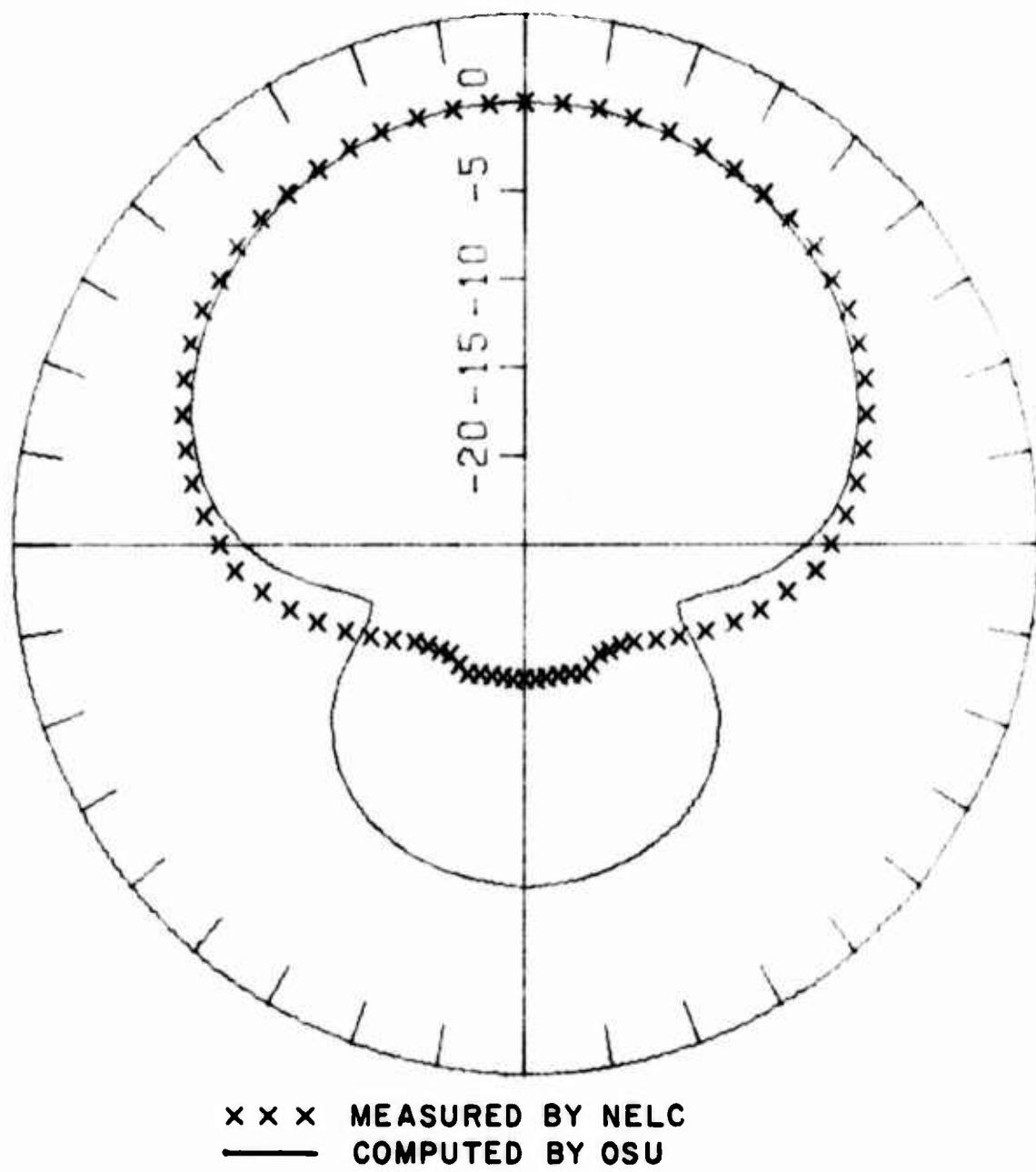


Fig. 23. Twin whip antenna pattern at 4.0 MHz ($\theta=85^\circ$).

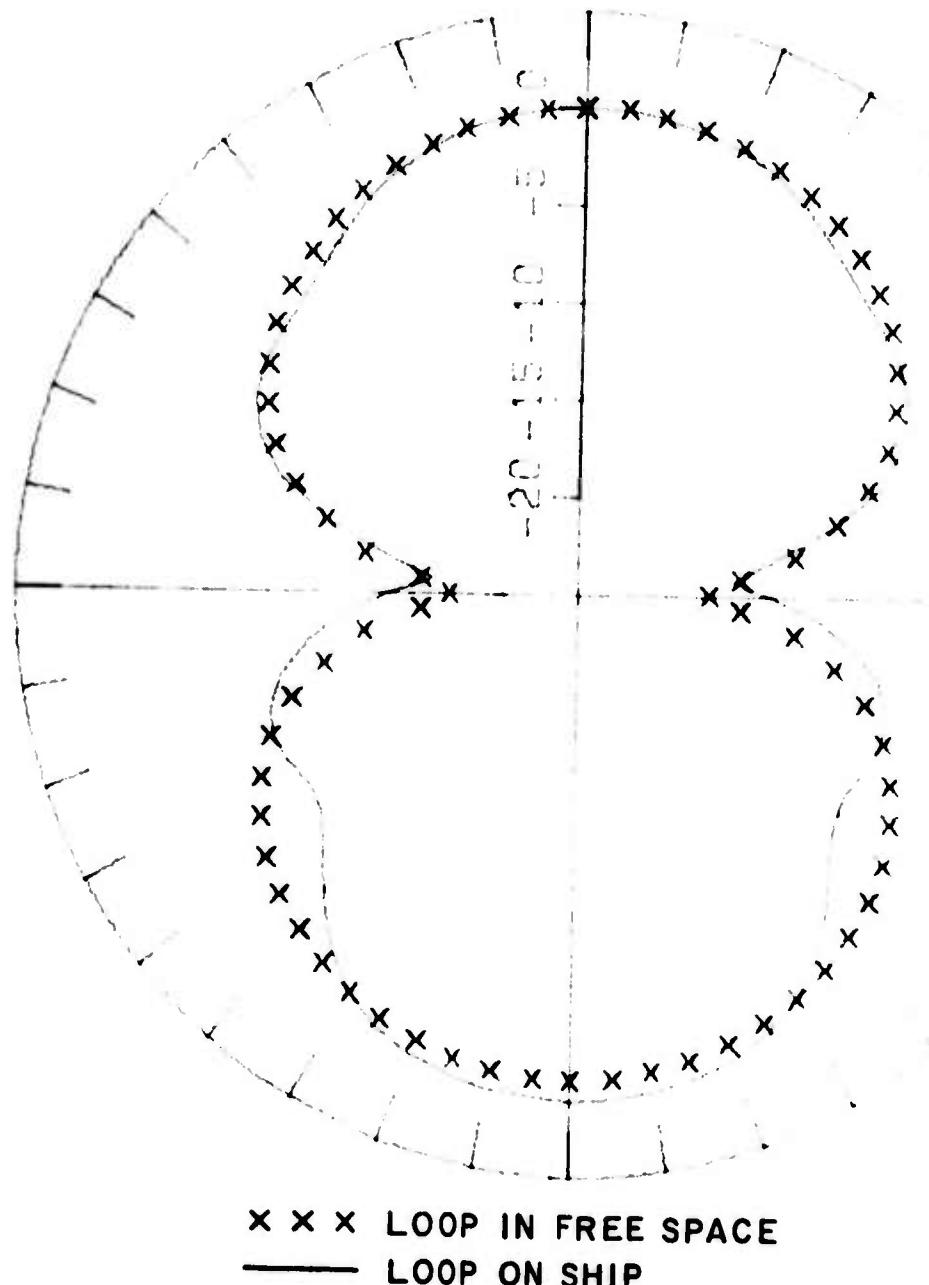
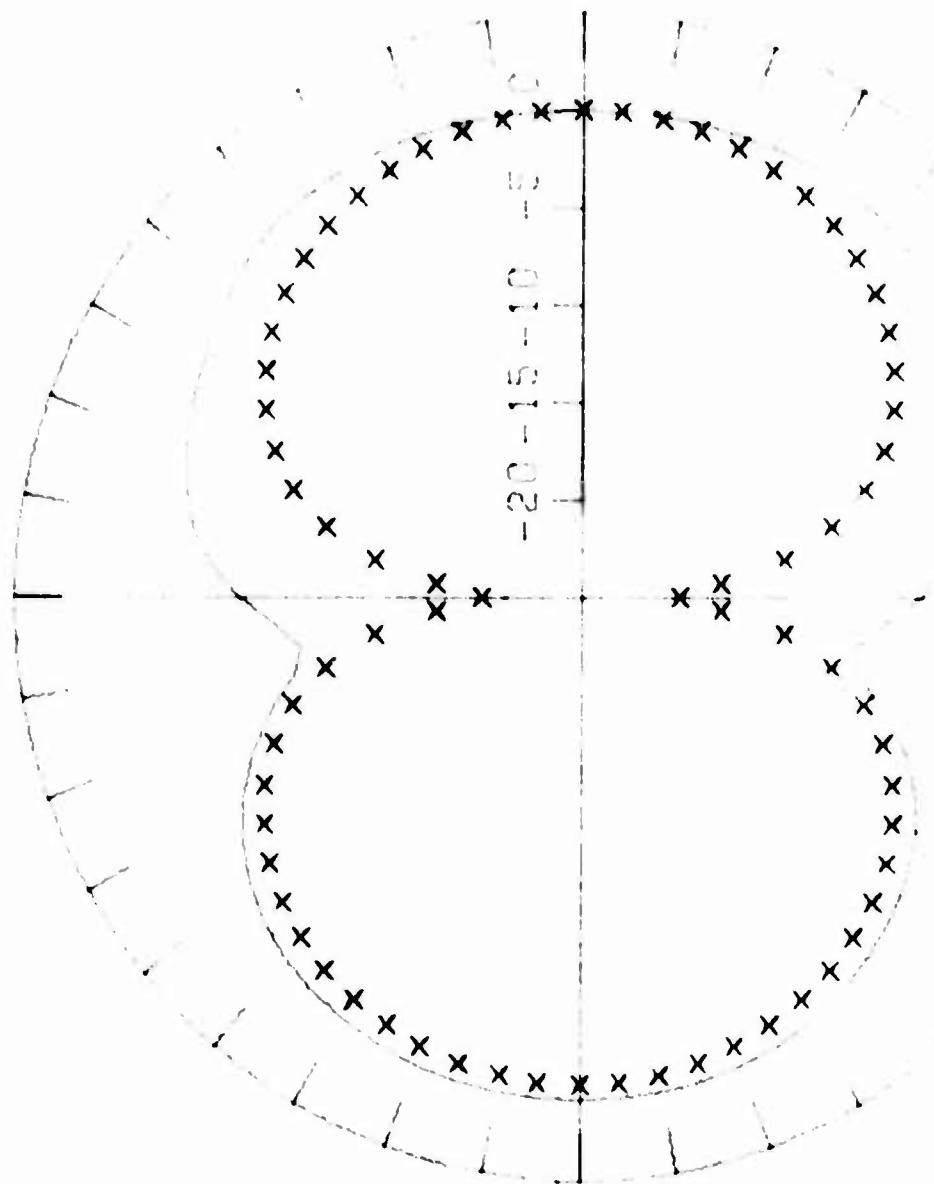


Fig. 24. Loop antenna pattern at 7.0 MHz ($\theta=85^\circ$).



× × × LOOP IN FREE SPACE
— LOOP ON SHIP

Fig. 25. Loop antenna pattern at 5.8 MHz ($\theta=85^\circ$).

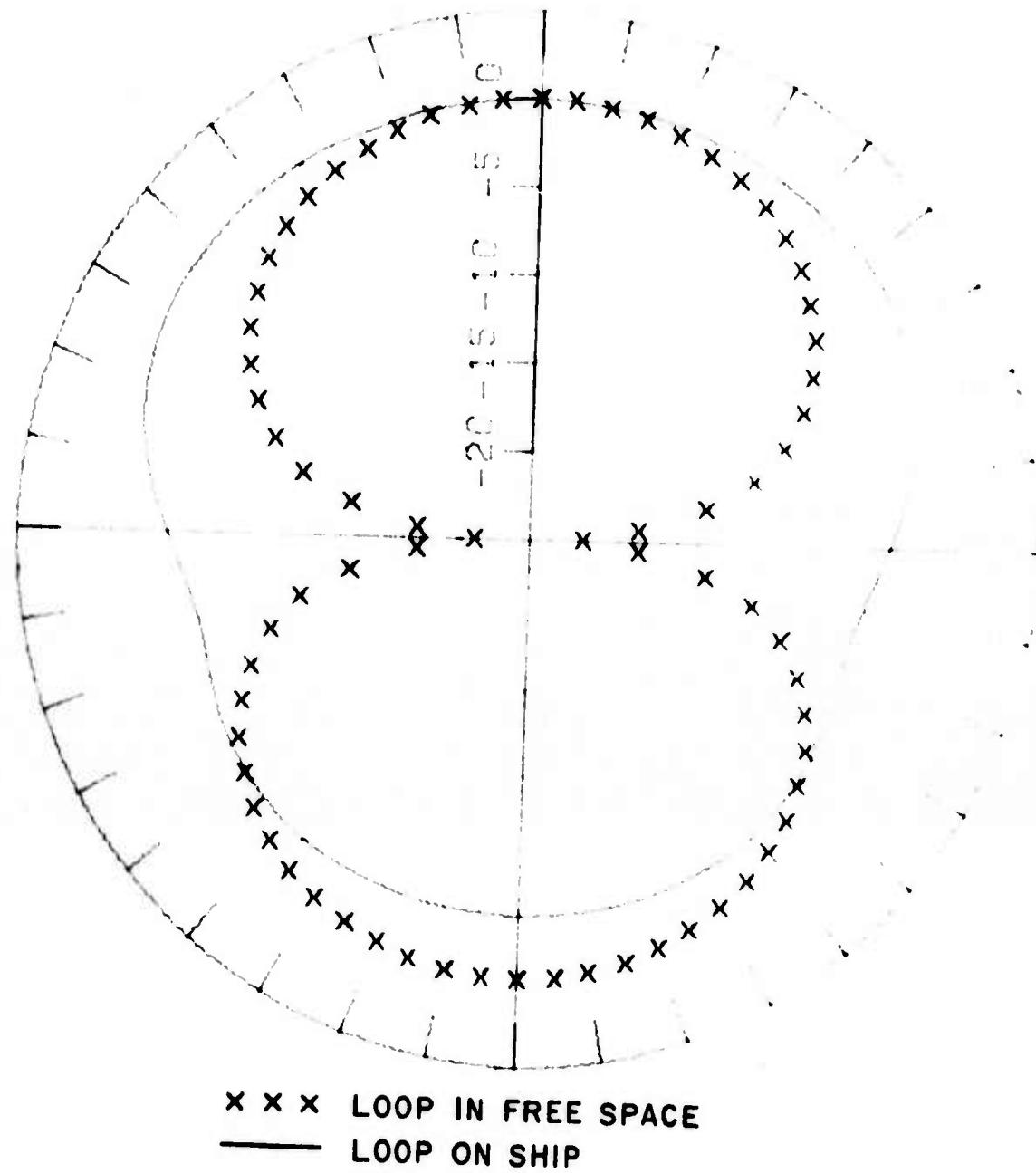


Fig. 26. Loop antenna pattern at 4.6 MHz ($\theta=85^\circ$)

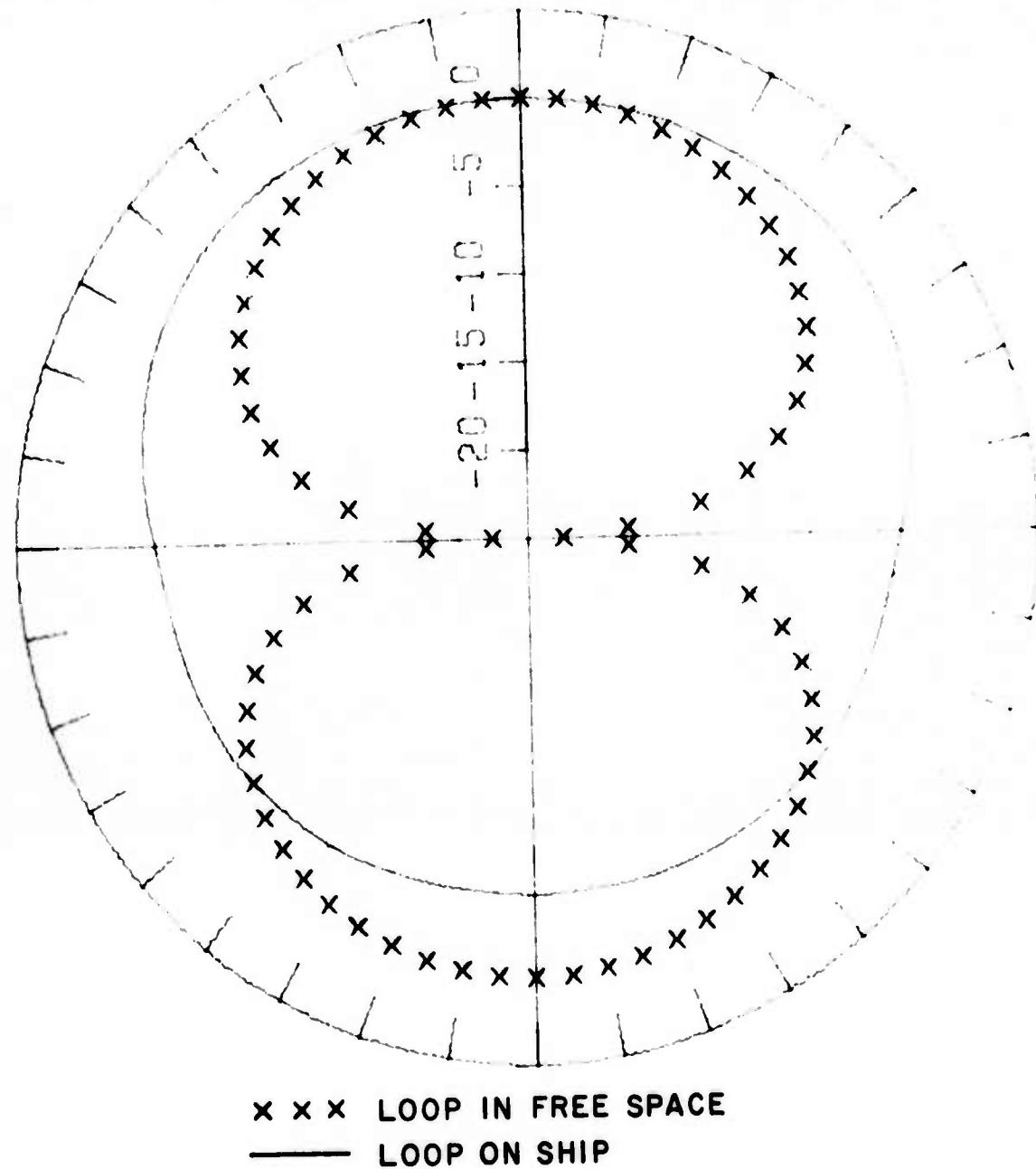


Fig. 27. Loop antenna pattern at 4.0 MHz ($\theta=85^\circ$).

V. RADIATION PATTERN MEASUREMENTS

Although it was felt that the radiation patterns of the MTL would be substantially those of a simple loop, some verification of this was thought desirable. Since the MTL is designed to work over a ground plane and this ground plane must be of considerable extent, it was not practicable to place both antenna and groundplane on a turntable, as would be done for pattern measurements at higher frequencies. The arrangement shown in Fig. 28 was therefore used. The antenna under test and its ground plane were placed on the ground. A pedestal having a horizontal drive axis was modified by extending a Fibreglas boom upwards from it at an angle of 45 degrees so that, at its highest point, the tip of the boom was vertically over the test antenna. A dipole probe and battery powered transmitter were mounted at the tip of the boom. The dipole could be mounted either parallel or perpendicular to the axis of rotation of the pedestal, but always perpendicular to the line between itself and the test antenna. Since the axis of rotation was not at ground level it was necessary to correct the patterns to obtain the true pattern. To facilitate this the patterns were recorded directly into a digital computer via an analog to digital converter. The computer then performed the necessary correction. Since it was not physically possible to move the test probe over the full 180 degrees because of ground interaction, the patterns were recorded over about 160 degrees. Linear extrapolation on a decibel scale was then employed to extend them to the full 180 degrees. Patterns were recorded for both polarizations of the dipole probe and for vertical planes through the test antenna at 22.5 degree intervals. All the patterns were then integrated, using the computer, to obtain the total power radiated. Using this result the patterns were replotted on a scale showing true directivity with respect to an isotropic radiator (Figs. 29 to 40). Polarization perpendicular to the rotation axis is shown by the solid line.

The patterns are essentially similar to those of a simple loop. Some asymmetry is seen which is partly due to the fact that the coil has a pitch; its turns are not exactly perpendicular to its axis. There is also some distortion due to the presence of the pedestal and surrounding buildings, though considerable care was taken to minimize these effects.

An efficiency measurement was also attempted by measuring the signal attenuation between a pair of identical antennas and applying the range equation using the directivity obtained from the pattern measurements. A lower efficiency was calculated than that obtained from the other methods described in this report. This result is not, however, considered to be a true measure of efficiency but rather points to the necessity of having a sufficiently large ground plane. With the cap in place, portions of the ground plane outside the 15 foot square area did not enter into the measurement of efficiency, although it has been demonstrated that reduced conductivity of the ground plane

due to grass growing through it has a substantial effect. In the case of the transmission measurement the ground plane around both antennas, beyond the 15 foot square area, consisted of grass alone. The metallic portion of the groundplane is still very small in terms of wavelength and is in no way representative of a shipboard environment. The higher signal loss observed in the transmission measurement is therefore believed to be a grass effect.

VI. CONCLUSIONS AND RECOMMENDATIONS

The multiturn loop (MTL) antenna has been developed for operation in the H.F. band from 2 to 30 MHz. A relationship between size and efficiency has been established which makes it possible for the antenna designer to optimize antenna efficiency for the specific environment available for the antenna. Where space is limited, such as with small ships or complicated environments, the MTL antenna becomes perhaps the simplest and most efficient and reliable radiator available. It's small size opens up many new possibilities, such as using multiple MTL's for reduced radiation hazard, for pattern control, for redundancy in case of damage, and for dedicated R.F. links.

The basic MTL concept has been shown to be a promising solution to problems where electrically small, low-profile H.F. antennas are desired. In order to broaden this concept and further develop the MTL, several areas for further investigation are recommended.

Although the band switching method used during this development period proved entirely satisfactory, additional work is recommended in this area to improve usability (remote switching) and maintain efficiency under normal shipboard operational conditions.

The tuning and matching system, although simple to use, would be improved by the development of an automatic matching and tuning system which would require only coarse presetting.

This system would require low-loss capacitive control elements only and very little additional space. It would relieve the operator of the requirement to control fine tuning.

Additional work is recommended to determine the effects on pattern and efficiency of loop placement on typical Navy vessels. This could be done by experimental modelling or by the numerical method mentioned in Section IV. The latter would be faster and less expensive. This work also could include an investigation of grounded versus "floating" loops.

In general, the MTL antenna concept looks very promising and should be developed further with full scale shipboard tests to more fully define it's characteristics.

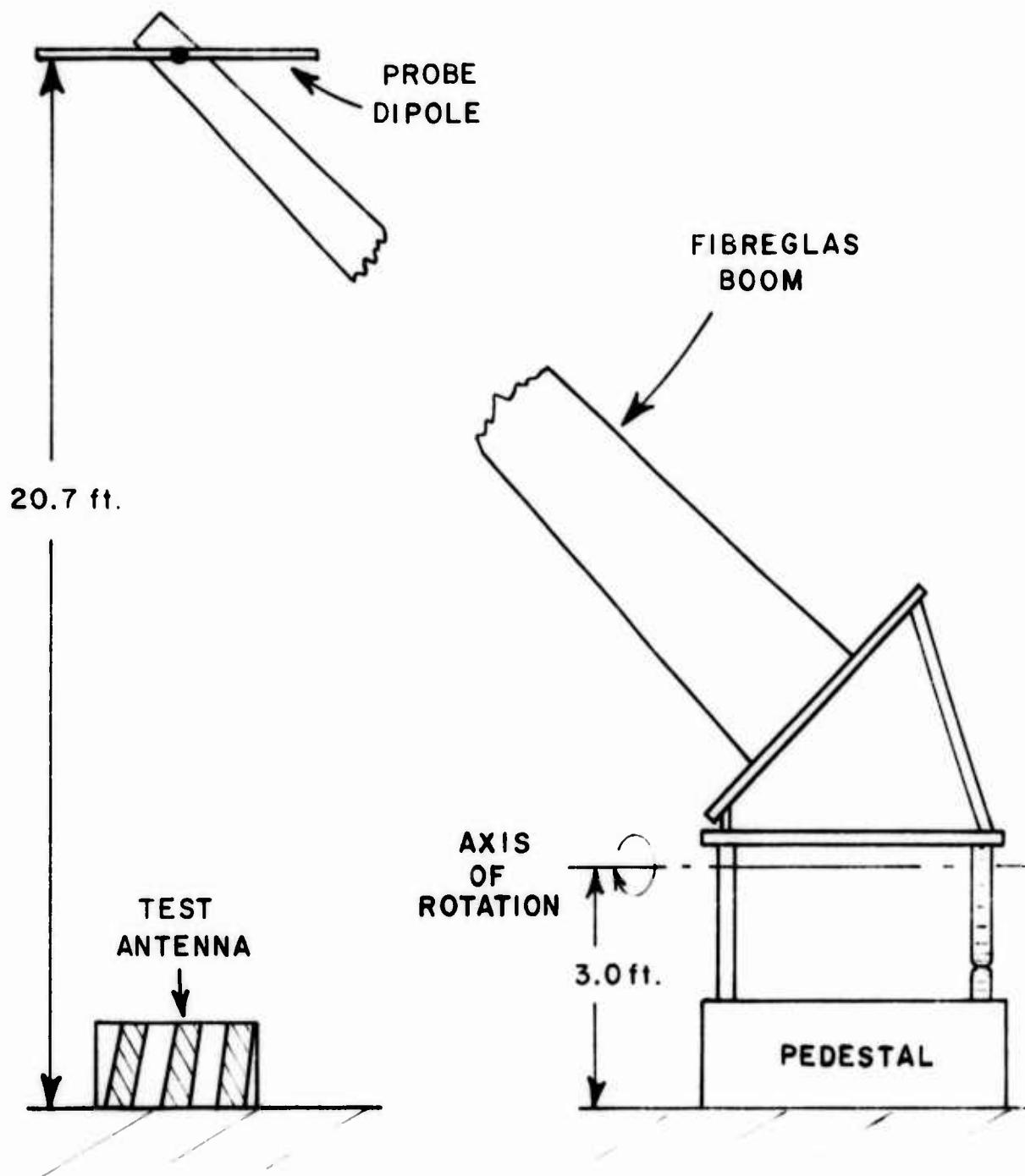


Fig. 28. Pattern recording system.

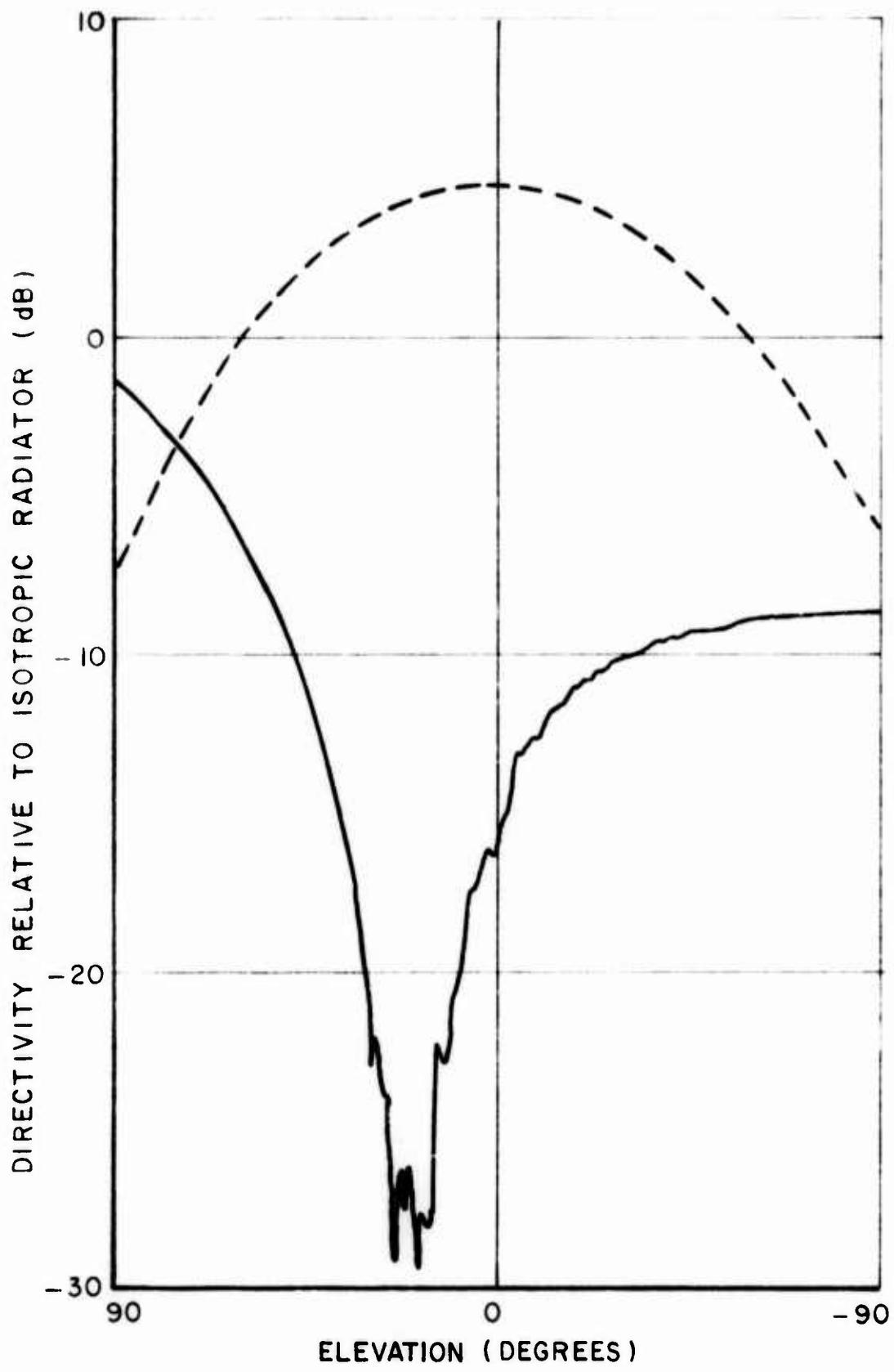


Fig. 29. Strap MTL antenna pattern at 3 mhz ($\lambda_z = 90^\circ$).

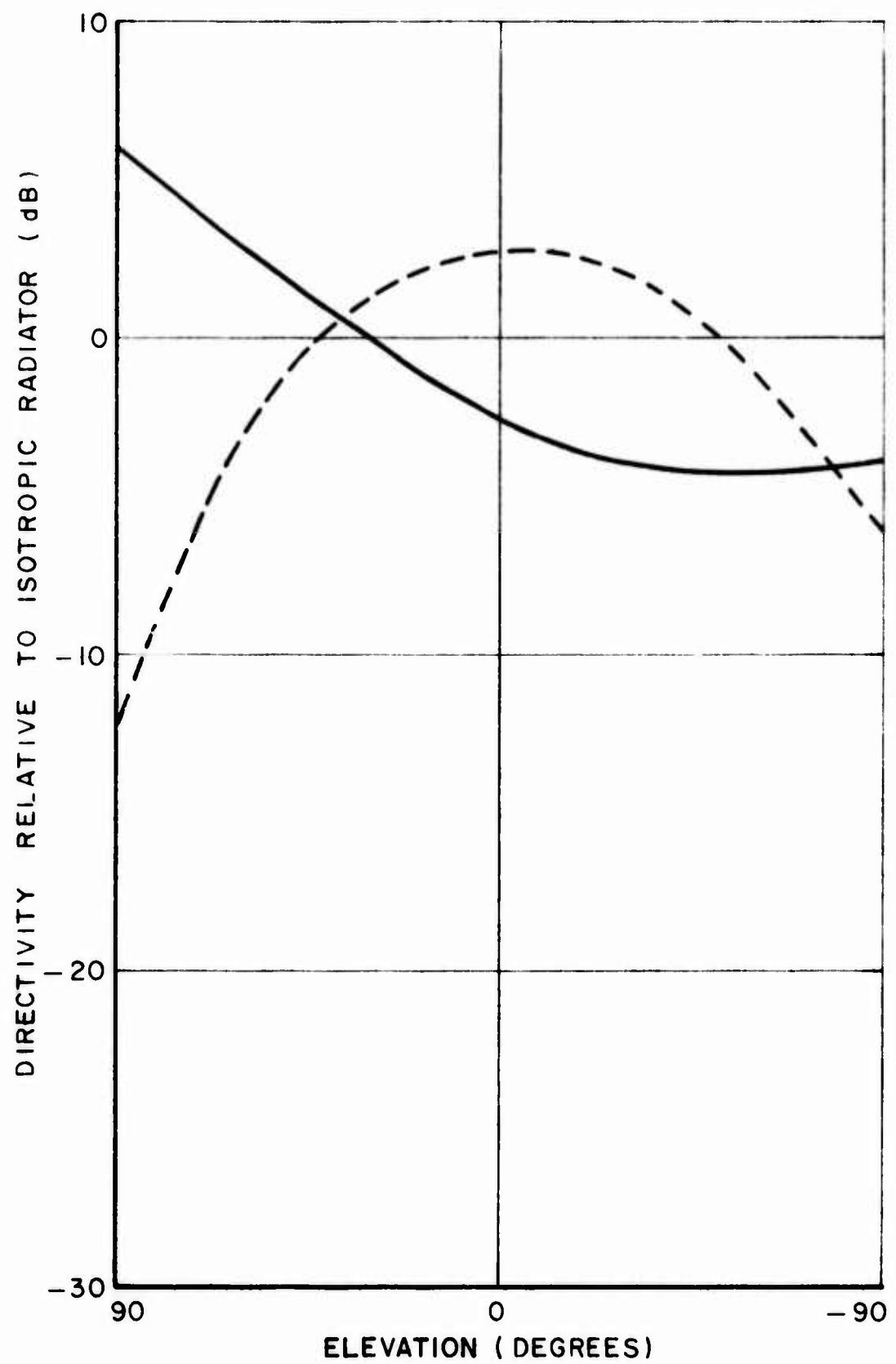


Fig. 30. Strap MTL antenna pattern at 3 mhz ($\Lambda_z = 45^\circ$).

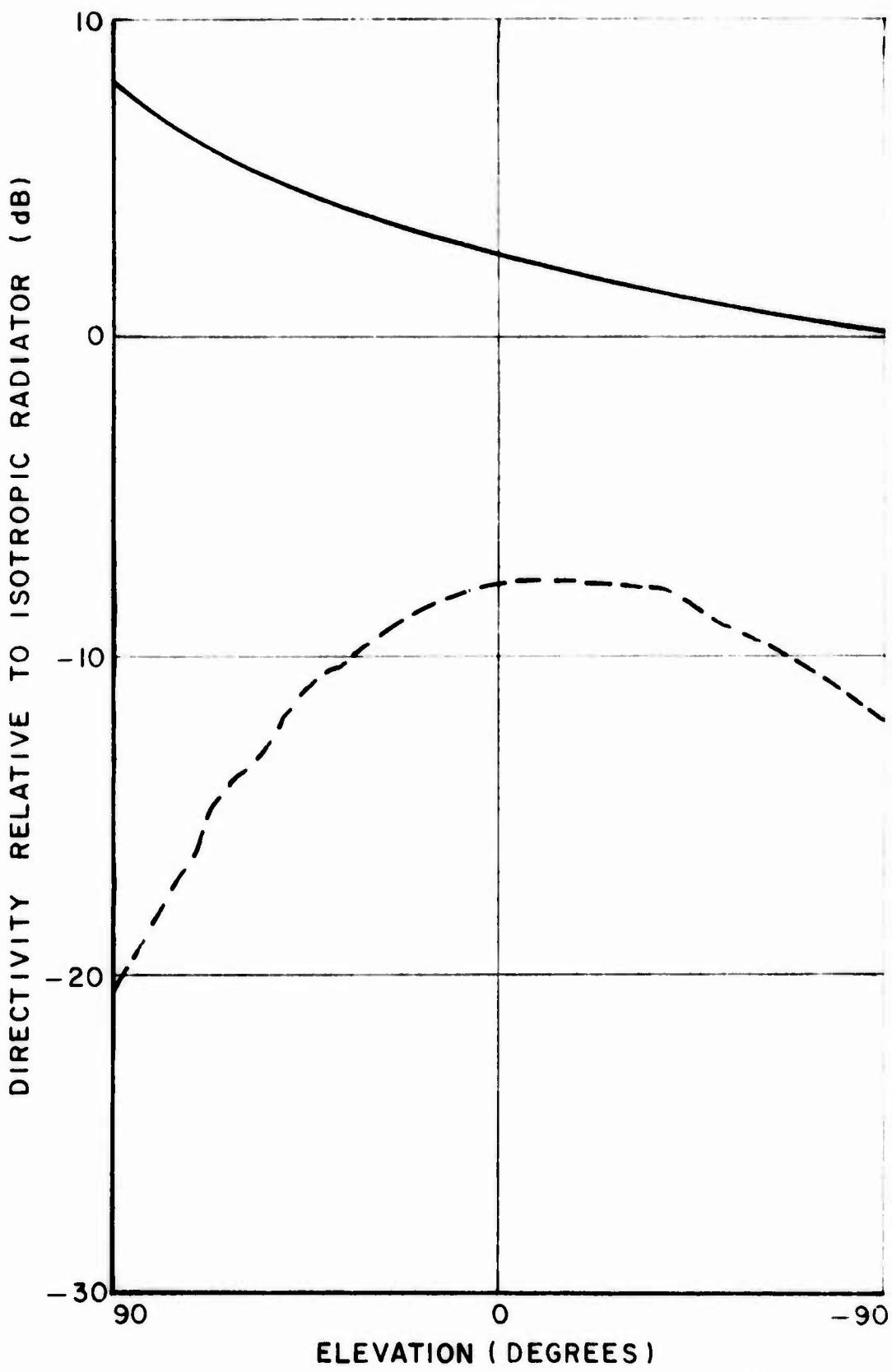


Fig. 31. Strap MTL antenna pattern at 3 mhz ($\Lambda_z=0^\circ$)

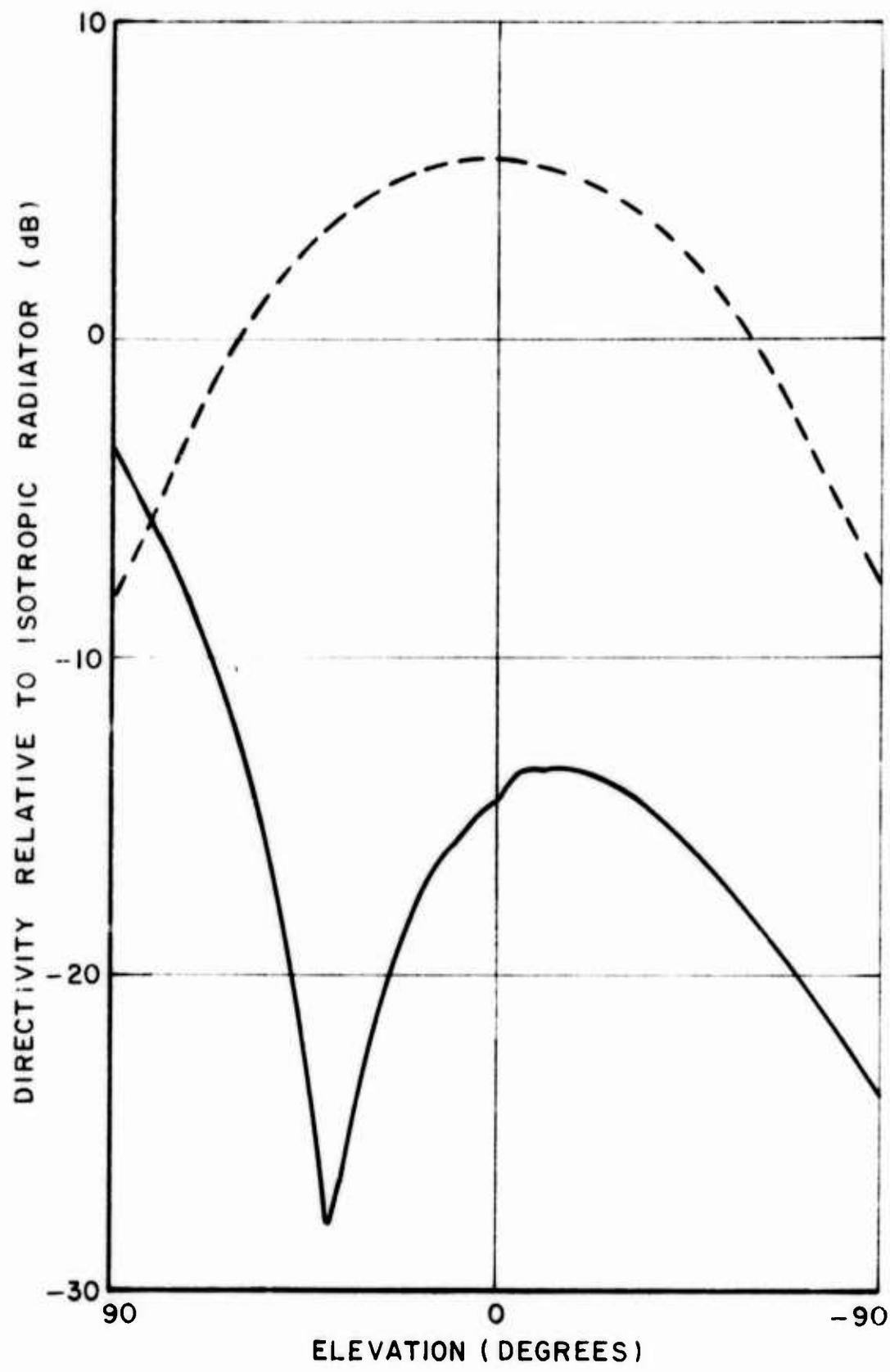


Fig. 32. Strap MTL antenna pattern at 6.2 mhz ($\lambda_z = 90^\circ$).

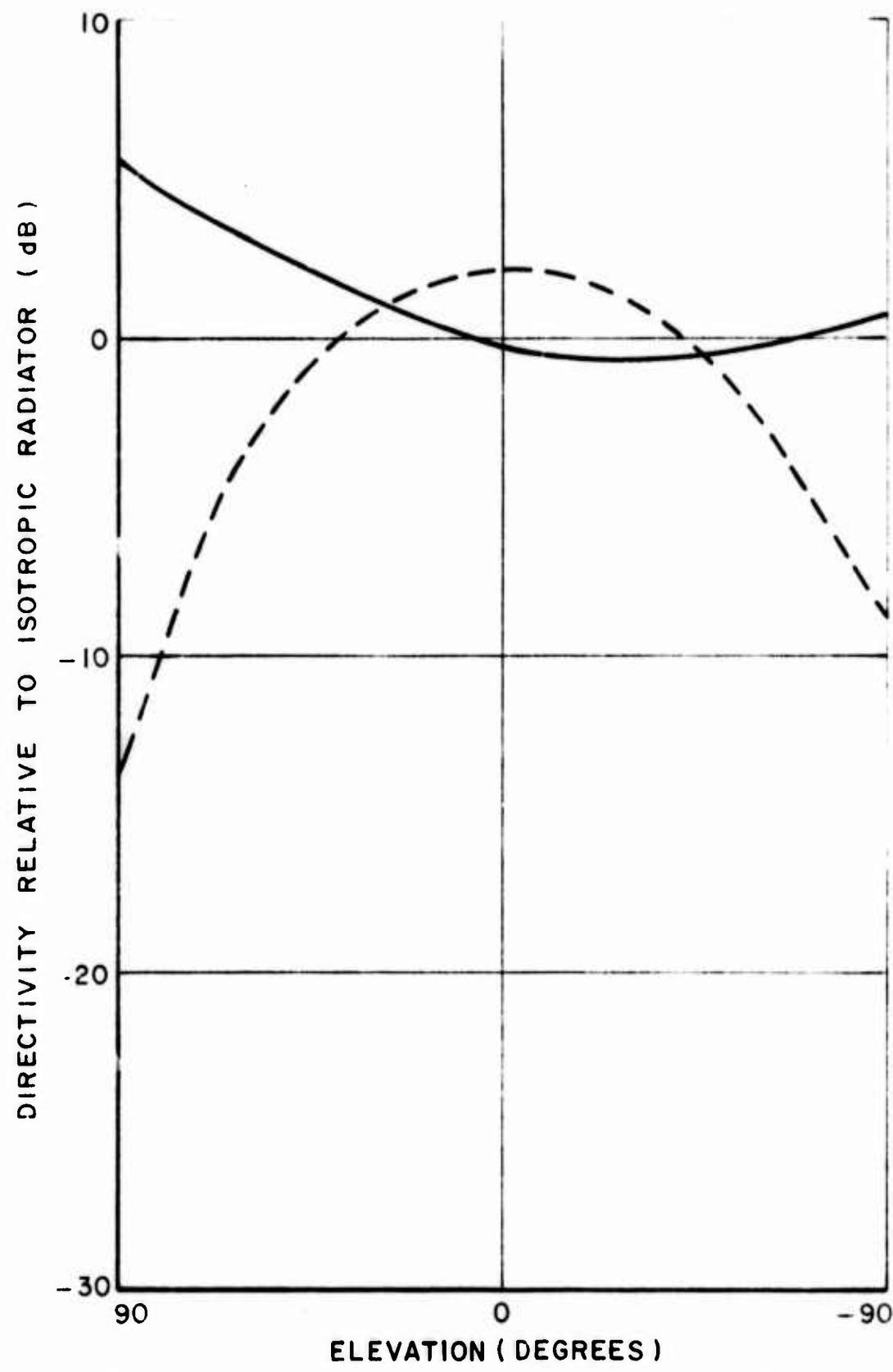


Fig. 33. Strap NTL antenna pattern at 6.2 mhz ($\lambda/45^\circ$).

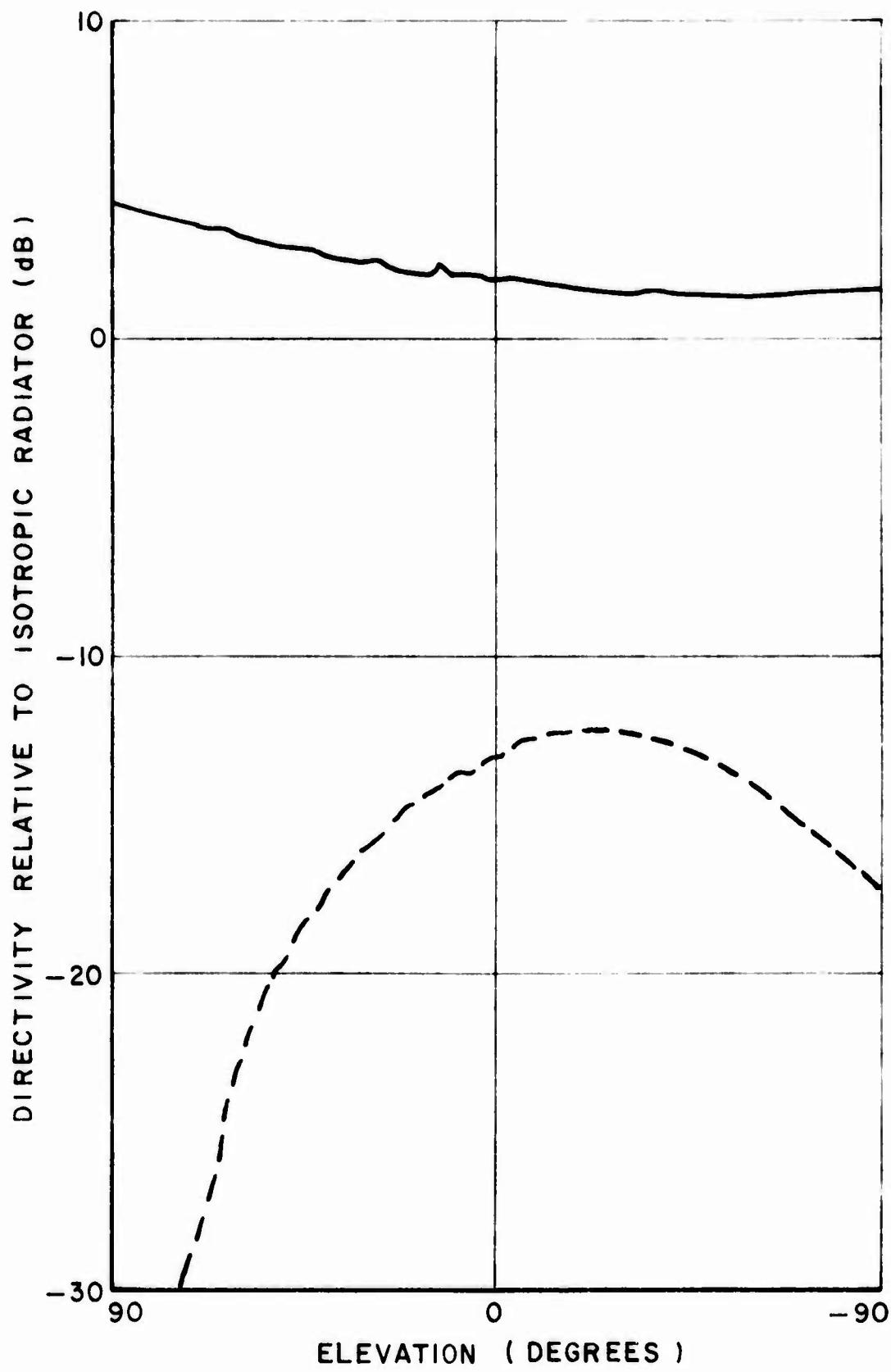


Fig. 34. Strap MTL antenna pattern at 6.2 mhz ($\phi_z = 0^\circ$).

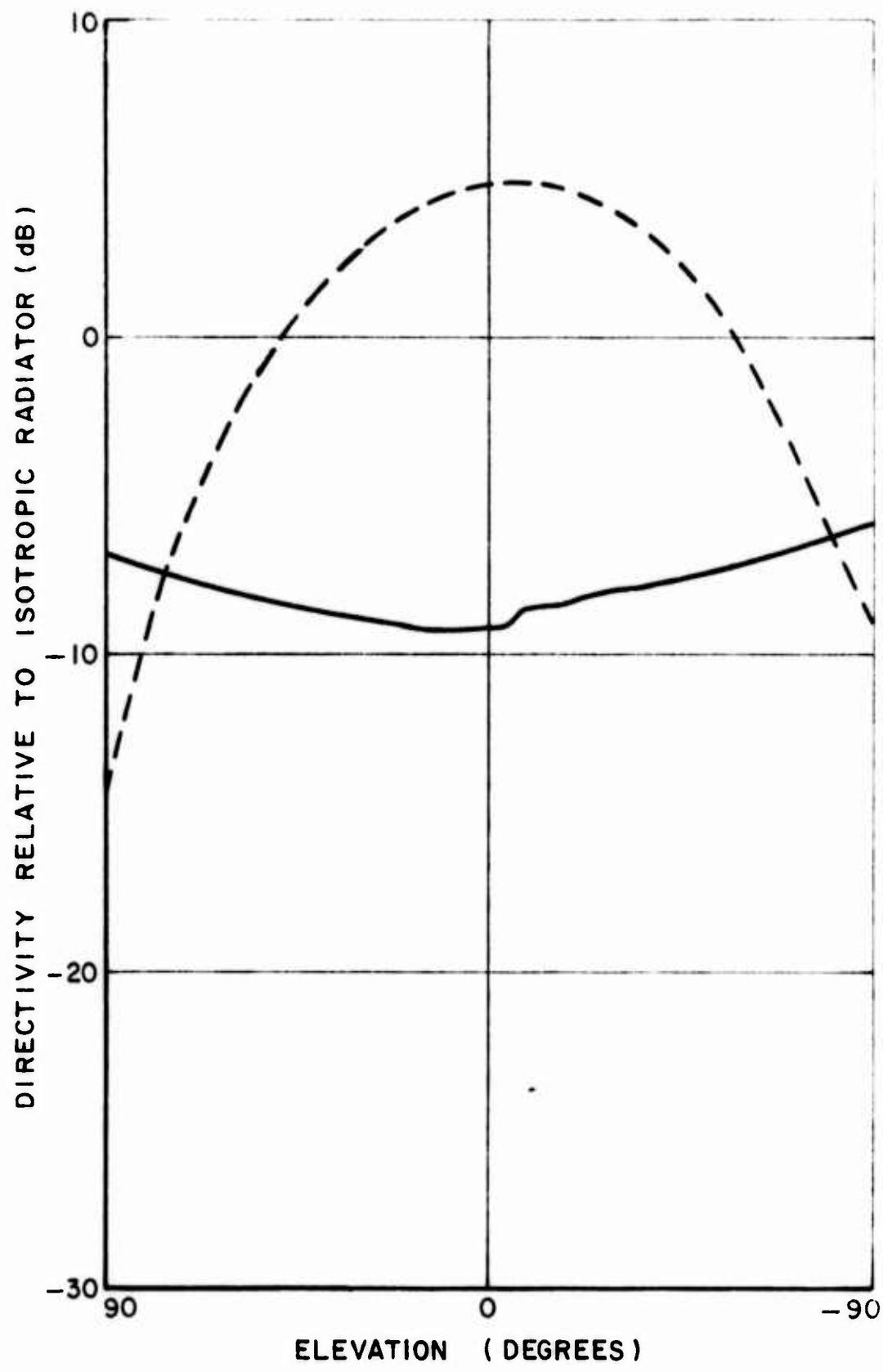


Fig. 35. Strap MTL antenna pattern at 12.7 mhz ($\Lambda_z = 90^\circ$).

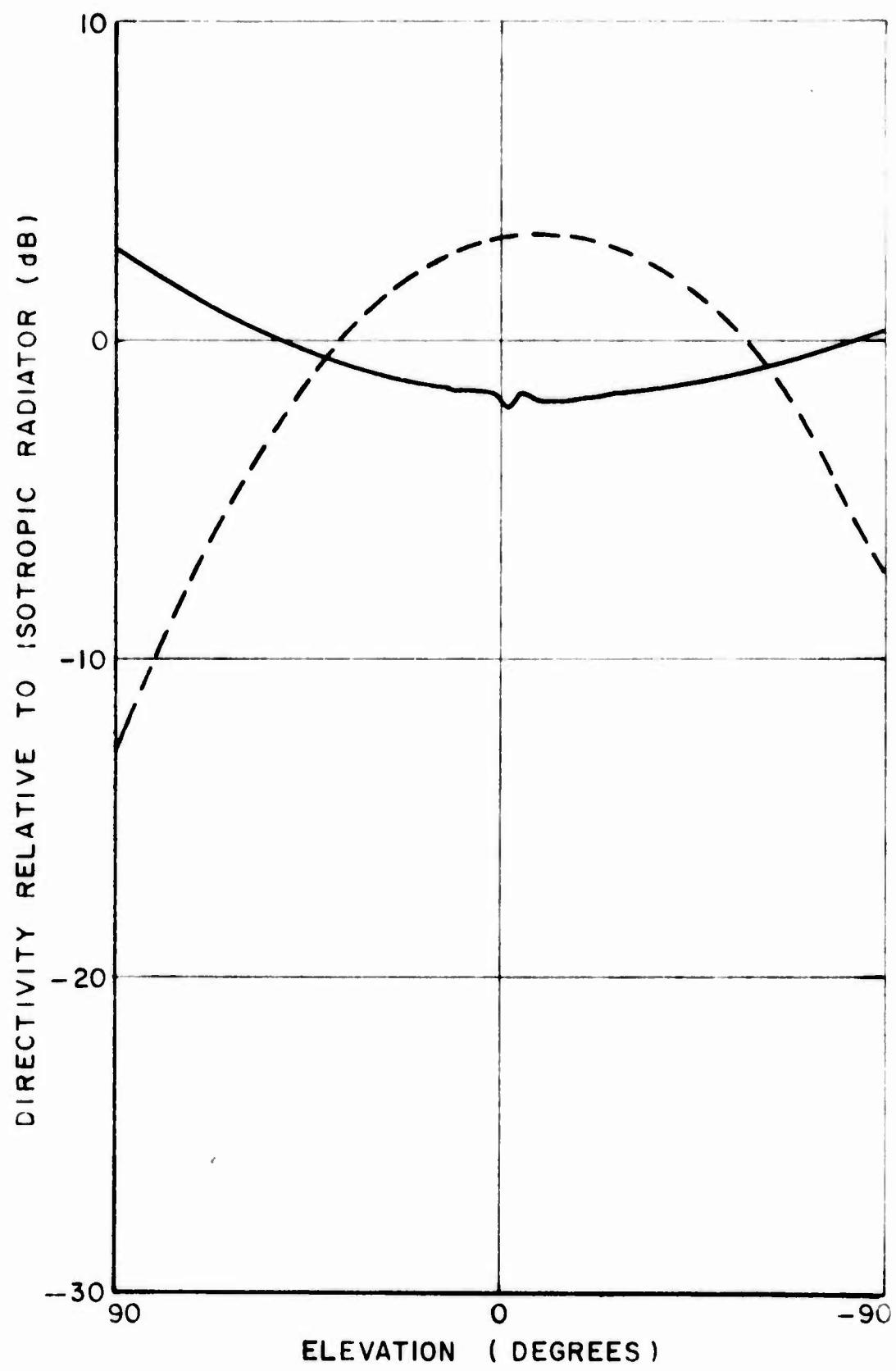


Fig. 36. Strap MTL antenna pattern at 12.7 mhz ($\lambda_z = 45^\circ$).

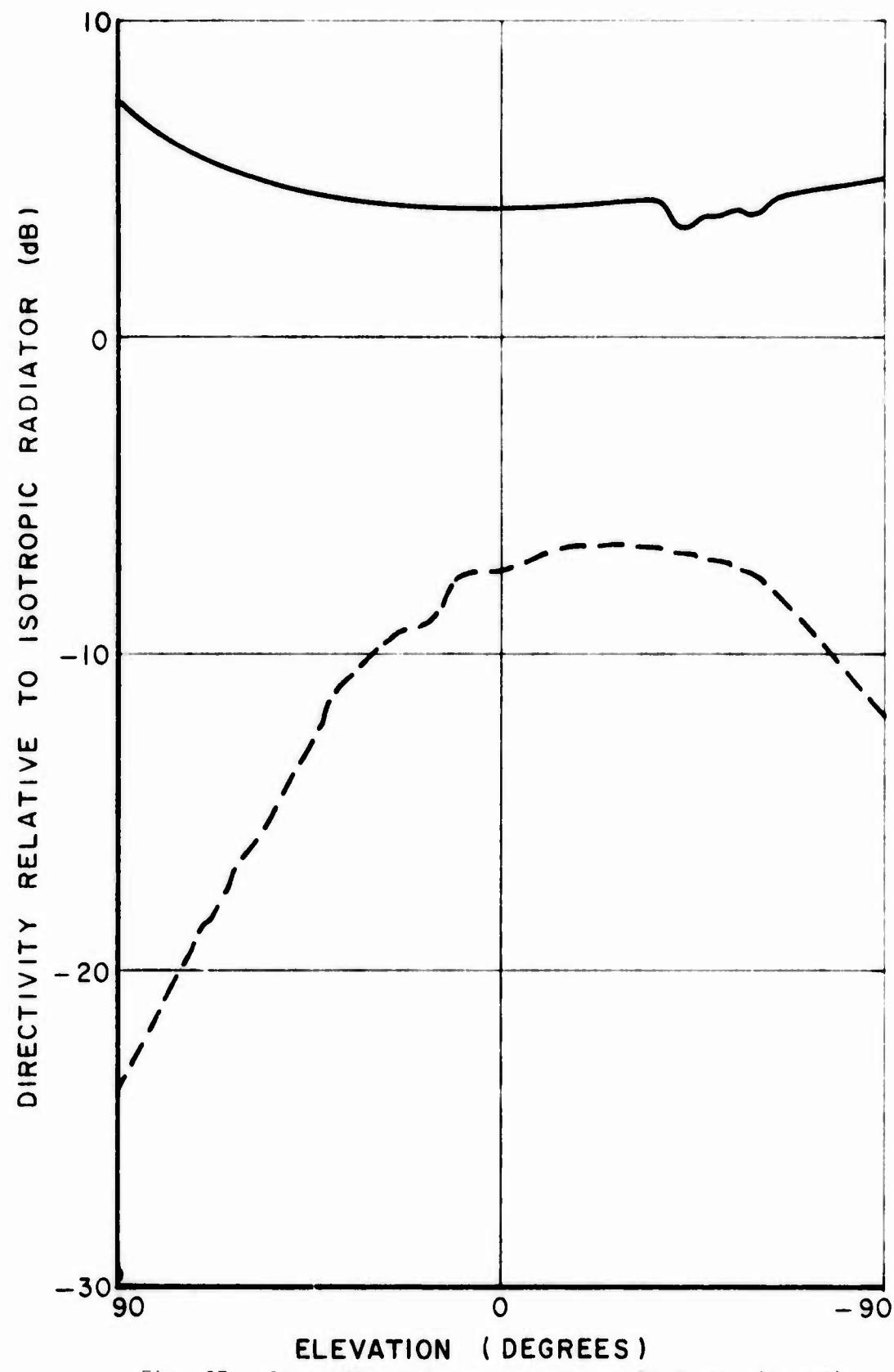


Fig. 37. Strap MTL antenna pattern at 12.7 mhz ($\Lambda_z=0^\circ$).

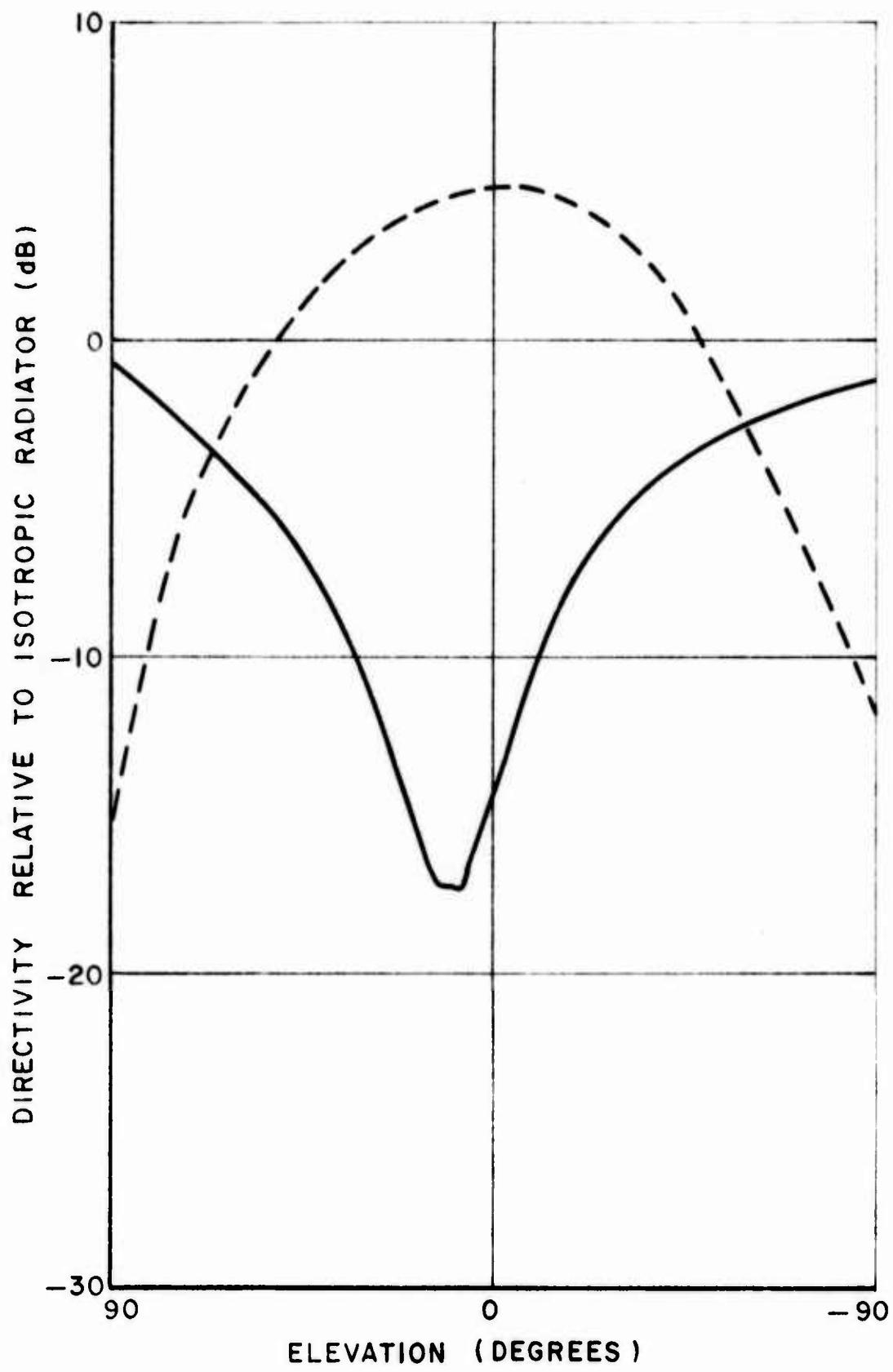


Fig. 38. Strap MTL antenna pattern at 29 mhz ($\lambda_z = 90^\circ$).

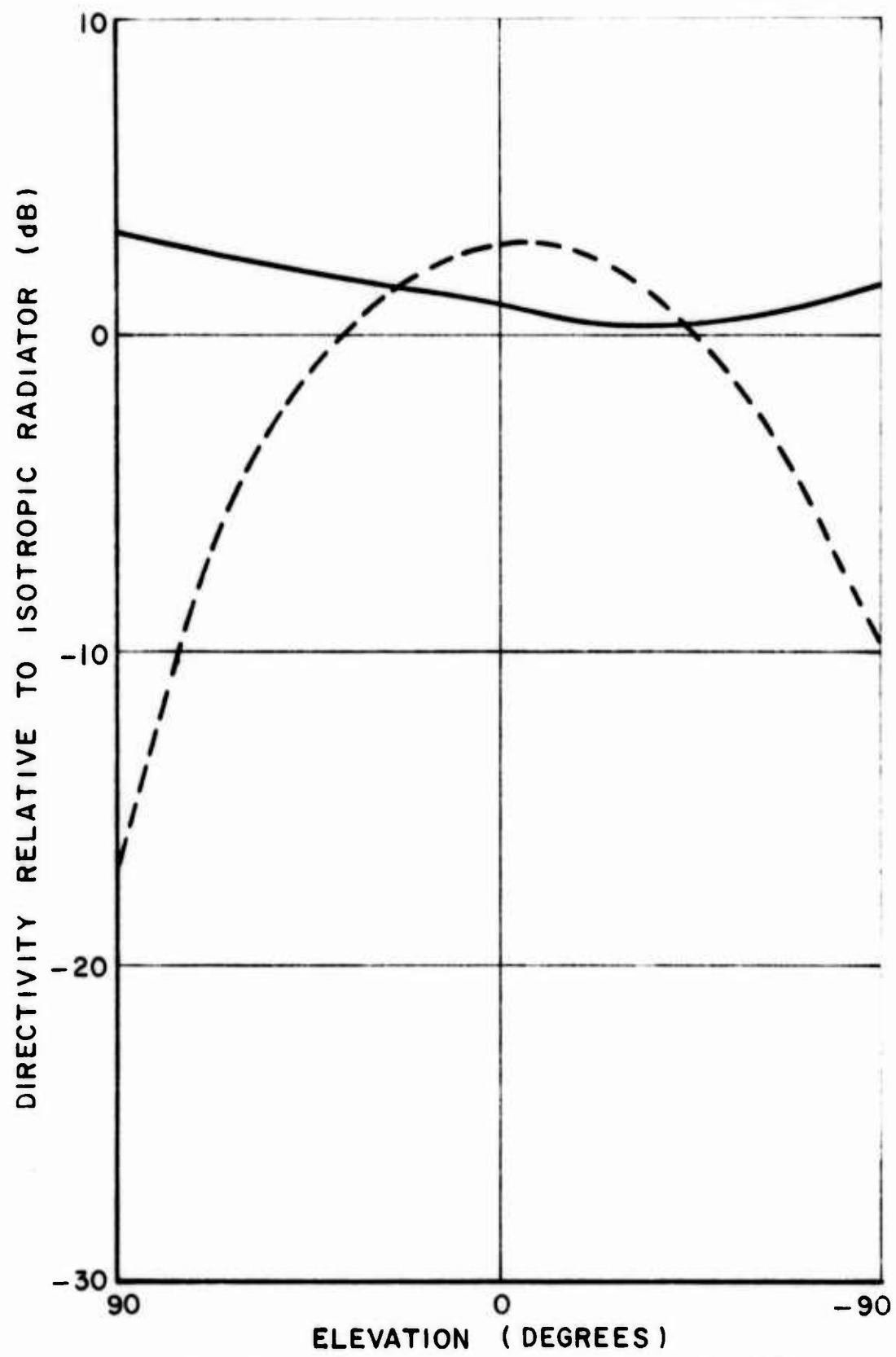


Fig. 39. Strap MTL antenna pattern at 29 mhz ($A_z = 45^\circ$).

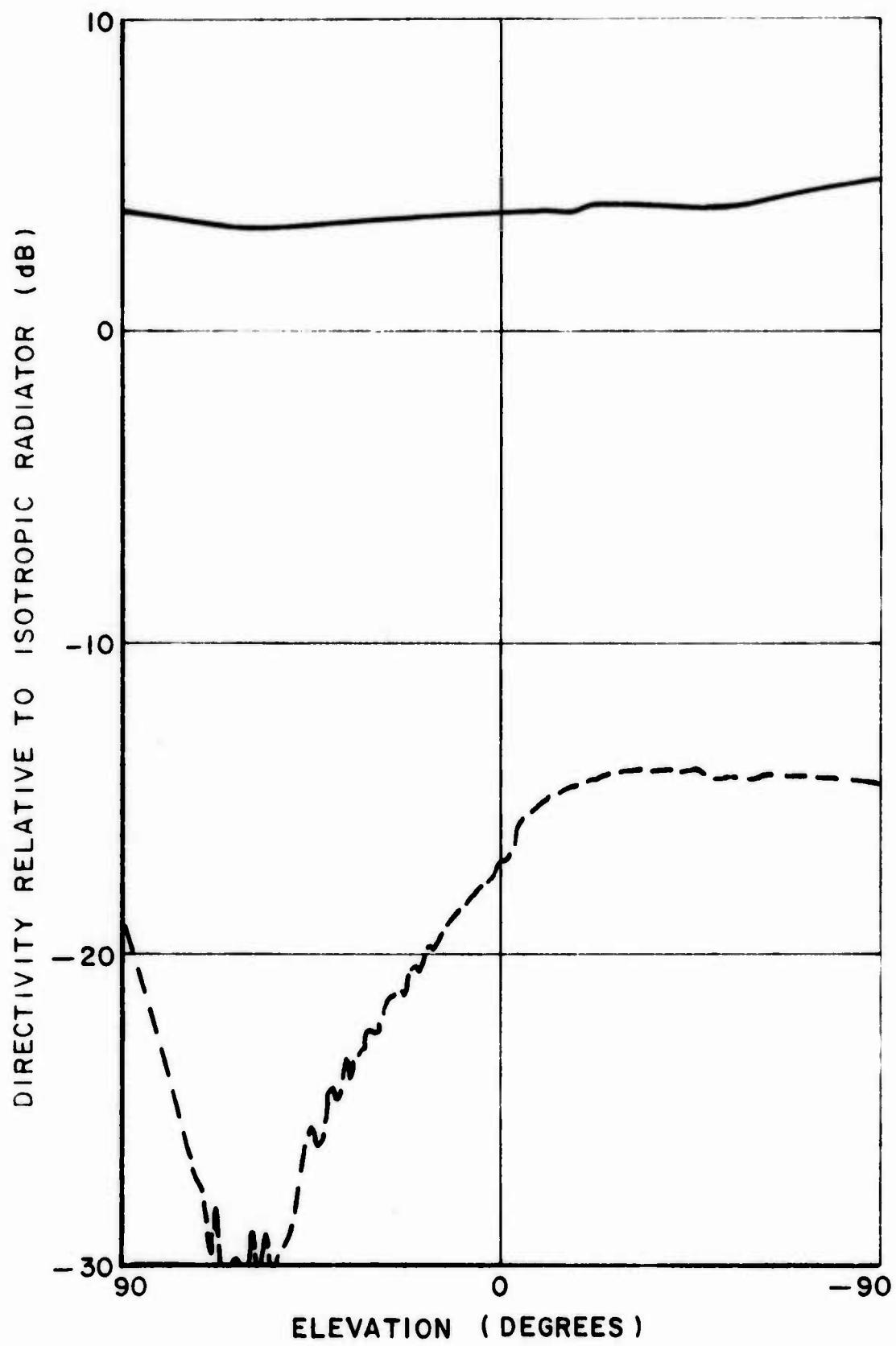


Fig. 40. Strap MTL antenna pattern at 29 mhz ($\Lambda_z=0^\circ$).

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